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VOLUME I

LESSONS IN LOGIC

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LESSONS IN LOGIC

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Archbishop of Baltimore

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To

His Eminence

James Cardinal Gibbons

Archbishop of Baltimore
Chancellor of the Catholic University of America
Fifty Years a Priest
Twenty-five Years a Prince of the Church

This Manual

is respectfully dedicated

by

The Author

PREFACE

The chief need of the teacher of logic in our high-schools and colleges is a text-book which will meet the peculiar requirements of the classroom. Many of the text-books now in use are written by men whose theory of the nature and value of knowledge, influenced as it is by false philosophical principles, is distrusted by the teacher who uses the books, and is not unlikely to upset the minds of the pupils. Others, while written on the soundest philosophical principles, take little or no account of pedagogical methods, and present the theory of logic without sufficient regard for the difficulties which beset beginners in this study.

The present text-book aims at supplying both these defects. It is based on the traditional scholastic theory of knowledge, and, wherever it touches on philosophical principles, the principles which it invokes in justification of the rules of logic are those of scholastic psychology and metaphysics. It aims at removing, as far as is possible, the technical difficulties of the study of logic, and tries to approach the problems of logic by the route which extended experience in the classroom has proved to be the easiest. If it has in any measure succeeded in this, it will have justified its appearance in a field which to some may seem already overcrowded.

The author wishes to record here his appreciation of the assistance which he has received from several teachers in our schools and colleges who have, for the last few years, used this text-book in typewritten form. He is especially grateful to Reverend Thomas Edward Shields, Ph. D., of the Catholic University of America, who has gone over all the copy, and to a Sister of Notre Dame, Professor of Logic and English at Trinity College, who has read all the proofs.

William Turner.

Washington, D. C.
June 1st, 1911.

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CHAPTER I

Introduction

Logical and Illogical. When we take up for the first time the study of logic we are, naturally, at a loss to give an accurate, scientific definition of it, to tell in so many words just what logic is, as a science, and as a method of mental training. Nevertheless, in a general way, so long as we are not obliged to put our idea in so many words, we have a notion of what logic is. We know when an assertion or an argument is logical. We can tell at once when a sentence or a series of statements is illogical. If we were to hear someone say "Shakespeare were an English man," none of us would fail to characterize the sentence as *ungrammatical.* If the assertion were made that "Shakespeare was a Scotchman," we should be able to affirm at once that the assertion is *false.* And if we came across the sentence "Shakespeare was an Englishman, because he was a Protestant" we should be able to pronounce it *illogical,* although one part of the sentence is certainly true, and the other very probably true, also. Again, if a critic of literature were to say "Shakespeare was a true poet, although he had no imagination," we should advert at once to the falseness of the second clause, and should be able, further, to point out that there is an inconsistency between the two clauses, and that the whole statement is, therefore, *illogical.* We know that a division of a subject or theme in composition is *illogical,* if the division is confused, or lacking in order. We say that a man who makes a promise and breaks it is inconsistent in his conduct, but we do not, on that account, say that he is illogical. But, if two assertions which he makes are inconsistent, we say that he is illogical. All this goes to show that, even before we take up the study of logic we know in a vague, indefinite way what logic is.

LOGIC IS A SCIENCE. When we begin to study logic as a branch of learning, we are told in the first place that *logic is a science.* We understand what that means. We know that physics, chemistry, biology, geology, are sciences. We know, too, though it does not occur to us at once, perhaps, that philology, of which grammar is a branch, is also a science, for it is the scientific study of language. We know that ethics is a science, the science of human conduct, and we know that theology is a science, the science of God and divine things. If, now, we compare the sciences that we know, we shall find that they all agree in this: they seek out the causes, laws, principles, or explanations of facts. The knowledge of facts is not in itself scientific. When, by actual measurement, we learn that a piece of iron is longer in summer than in winter, we know a fact. When we learn that the cause, or explanation, of that fact is heat, our knowledge begins to be scientific. *Logic, then, is a science in so far as it seeks the causes or explanations of the rules which it lays down.* And in this respect it does not differ from any of the other sciences which we have already studied.

Logic Treats of Thoughts. In the next place, if we reflect how the various sciences already known to us differ among themselves, we shall see very soon that they *treat of different things.* Astronomy treats of the stars, botany treats of plants, grammar treats of words and their forms, zoology treats of animals, and so forth. The most general assertion, then, that we can make about the science of logic is that *it treats of processes of thought.* In this it differs from astronomy, botany, grammar and zoology, just as these differ one from another.

Thoughts, however, may be unkind, selfish, uncharitable. These, we see at once, are moral or ethical, qualities of thoughts, and the consideration of them belongs to ethics. Or, we may consider the origin of our thoughts, studying how they arise in the mind; we may consider the influence which one thought has on another, or the influence they have on character, observing that generous thoughts make a person noble and large-minded, while selfish thoughts tend to make a person ignoble and narrow-minded. The origin of our thoughts, the influence of one thought on another, and the effect of thoughts on character—all these belong to the science called psychology, which is the science of the soul and of its conscious states. It belongs to logic to consider whether our thoughts are clear and orderly, whether they are consistent with one another, and whether, if one thought is said to follow from another it follows validly, or soundly. For instance, we are told that on one occasion Pericles said, "My son's dog, Azor, rules my son, he rules his mother, she rules me, I rule Athens, Athens rules the world; therefore, the dog Azor rules the world." A historian would be interested in finding out whether Pericles really said this. A psychologist would wish to know in what frame of mind Pericles made this series of statements. He would like to know if it was meant as a jest, or what was the purpose of the speaker if it was meant to be serious. Further, the psychologist would examine the different states of mind referred to; the boy's interest in the dog, the mother's affection for the boy, the husband's love for his wife, the popularity of the statesman among his fellow-citizens, the authority which Athens for a brief period enjoyed, owing chiefly to her intellectual and cultural supremacy. For the logician, however, the point of interest is the sequence of the conclusion from the statements that go before it. Does it follow that because Azor rules Pericles' son, etc., therefore, Azor rules the world? Again, when a speaker or a writer declares at one time that "Courage is a virtue" and at another that "Discretion is the better part of valor," the psychologist is interested in the analysis of courage, virtue, discretion, valor; and, possibly he may inquire into the motives which may have influenced the speaker or writer to veer around from the advocacy of courage to the inculcation of discretion. The logician is interested in the question: Are the two statements really incompatible, or inconsistent?

It would be easy to say that the difference between psychology and logic lies in this that psychology merely observes and studies mental processes or thoughts, while logic directs them. The assertion, however, is not entirely accurate; because psychology, as everyone who has studied it knows, directs our thoughts also. It directs them towards mental development, self-control, character formation, etc. Logic directs them ultimately towards mental development, the formation of character, and self-control; but *its immediate purpose is to direct them towards clearness, order, consistency, and validity.*

LOGIC NOT CONCERNED WITH ALL OUR MENTAL PROCESSES. The difference between psychology and logic will be made clearer if we proceed now to the next assertion concerning the subject-matter of logic. *Logic is not concerned with all our mental processes.* Feelings, such as the feeling of pain; emotions, such as anger; volitions or acts of the will, do not come directly within the scope of logic, because, although they are states of mind, and often influence our knowledge, they are not themselves knowledge-states, or cognitive states, as they are called. The knowledge-states, or processes of knowing, are divided by the logician into three kinds:

I. *Mental Images.* These may be the simplest kind of impression, such as the impression of whiteness which I receive when I look at the paper before me. Or they may be percepts made up from a number of impressions, such as my percept of the apple on this table. Or they may be concepts, or ideas properly so called, built up by a comparison and contrast of various percepts, such as my idea of an apple, a triangle, a tree.

II. *Judgments,* which are knowledge-states more complex than simple mental images, because they imply an act pronouncing an agreement or a difference, as when I judge "This apple is red," "No triangle is a four-sided figure."

III. *Reasonings,* or the most complex kind of knowledge-states, by which we pass in thought from some judgments already known to be true to a new judgment. When, for example, I say "Blue is a physical quality, because it is a color," the sentence expresses a complex mental state. Fully expressed, the process would be "All colors are physical qualities; blue is a color; therefore, it is a physical quality." It is perfectly evident that there are three stages in this process of reasoning; the first is that by which we judge that "All colors are physical qualities," the second, that by which we judge that "Blue is a color," and the third process takes place when from these two we pass to the new judgment, "Therefore, blue is a physical quality." The whole process is one of reasoning. It implies several judgments. If we now analyse each judgment, we shall find that it implies the presence in our minds of mental images. For example, "Blue is a color" implies the notion or mental image of "Blue" and the notion, or mental image, of "Color."

Reasoning, judgment, and mental images—these are the knowledge-states of mind with which logic is concerned. Psychology is concerned with all states of mind, logic with these alone. Moreover, logic is concerned with reasoning from the point of view of soundness or validity. It is concerned with judgments from the point of view of consistency. It is concerned with mental images from the point of view of clearness and order. We may, then, define logic provisionally by saying that *Logic is the science of reasoning and of the thought-processes implied in reasoning, with reference to validity, consistency and order.*

Logic as a Preventive of Error. Some prefer to define logic as a preventive of error. It is, indeed, a science which teaches us how to avoid error, inconsistency, and confusion. Those who define logic in this negative way do so for the purpose of discriminating between it and the Art of Discovery. It is not the business of logic to teach us how to find arguments or how to discover new truth. Its business is to teach us how to test arguments and to examine the soundness of the processes by which truth is discovered. It lays down the rules of right reasoning, without undertaking to furnish reasons on any specific topic, just as rhetoric lays down the rules for the use of figures of speech, but does not undertake to furnish metaphors, similes, etc., to the essayist, the orator, or the poet. Both the negative and the positive method of defining logic give us the same result. The difference between the science which enables us to test the validity of an argument and the science which enables us to detect the invalidity of an argument is only a difference of words.

Logic is an Art. There has been a good deal of discussion among writers on logic as to whether it is an art or a science. Here, again, the difference is largely a difference in the use of words. Logic is a science, in so far as it not only sets forth an organized system of rules for the direction of the mind, but also traces those rules back to their principles, to their foundation in the nature of the mind and of truth. That is to say, it is a science, in so far as it gives reasons for the rules which it prescribes. It is also an art. The only difference between it and the recognized arts of music, painting, etc., is that, while they prescribe the rules or principles which govern external actions, logic lays down the rules which govern the internal action of the thinking mind. Logic as a science may be compared with the theory of medicine, the theory of music, or the theory of perspective; logic as an art may be compared with the practice of medicine, the art of music, or the art of drawing. Being an art as well as a science, logic is essentially directive, not purely speculative.

Definition of Logic. We may, therefore, sum up the foregoing statements regarding logic, and define it as "*The science and art which so directs the mind in the process of reasoning and subsidiary processes as to enable it to attain clearness, consistency, and validity in* *those processes.*'"[1] Clearness in the arrangement and definition of our ideas and other mental images, consistency in our judgments, and validity in the processes of inference—this is the aim of the student and teacher of logic. In what, precisely, clearness, consistency, and validity consist will be discussed in subsequent lessons.

Logic and Rhetoric. It is necessary at this stage to indicate the difference between logic and rhetoric. They have this in common, that they both lead to conclusions. Yet, they have important points of difference; logic has to do with the thinking faculty, rhetoric deals also with the emotions; logic aims at convincing, rhetoric aims at persuading; logic is concerned with the attainment of truth, rhetoric incites to action. Rhetoric is concerned chiefly with the expression of thought; logic, chiefly with the thought itself. Rhetoric has its legitimate uses. It is not enough to convince; one must add persuasion to conviction in order to render conviction vital and operative; and in order to do this it is necessary to call into play the emotional and volitional, as well as the purely cognitive faculty. It is only when rhetoric trenches on

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[1]St. Thomas, in his commentary on Aristotle's Logic (*In Post. Anal.*, Lect. I) writes: "Ars quaedam necessaria est, quae sit directiva ipsius actus rationis, per quam, scilicet, homo in ipso actu rationis ordinate, faciliter, et sine errore procedat. Et haec ars est logica, idest rationalis scientia."

the domain of logic and clashes with the rules of right reasoning that the use of it becomes an abuse. When it persuades by deceiving, then it ceases to be a legitimate means of persuasion and becomes sophistry. Thus it is clear that logic controls rhetoric. For, while it is true that in the expression, oral or written, of our mental processes, especially when we have in view the conviction or persuasion of others, logic without rhetoric is lifeless and inefficient, it is true at the same time that when rhetoric ceases to conform to the requirements of logic it serves the cause of error and not the cause of truth.

THE USES OF LOGIC. In recommending the study of logic, that is to say, in pointing out its usefulness, one should, above all, be logical. It does not become the teacher of logic to advocate this one science as the universal remedy for all diseases of the mind. One has only to consider what logic is in order to realize that it has its uses. But those uses should not be exaggerated. Logic does not teach men how to reason. Before the science was ever heard of, men reasoned, and reasoned well. And, even in our own time, men reason accurately and validly about things in which they are greatly interested, although they may never have had a training in logic. Children, for instance, will reason with remarkable skill and accuracy about their games, or in defence of their own conduct when called on to account for it. A special knowledge of the subject under consideration (the mechanic's knowledge of machinery, the farmer's knowledge of matters pertaining to agriculture, etc.), or general tact, skill, or insight enables a person to reason rightly and to draw conclusions which are perfectly valid. Such a person, however, cannot justify the process of his own thought. He cannot give a reason for his reasoning. And the consequence is that his success as a reasoner is irregular and haphazard. Outside the limited range of his special vocation he is as liable to err as he is to think correctly. Logic, by giving us an insight into the universally valid laws of correct thinking, helps us to reason rightly; it removes correct thinking from the region of the fortuitous, and makes it a matter of rule and the correct application of rule. One who has learned to test mental processes by universally valid laws should be able to reason validly on any subject, no matter how remote from his daily vocation or his dominant interest, provided he know enough of the subject to understand the terms of the problem. Again, to consider for the moment the negative side of the matter, "Common sense" will enable us to detect a fallacy in such an argument as the following: "This boy is both mischievous and clever. Therefore, all mischievous boys are clever." Common sense, however, will not tell us why the argument is invalid, so that when the same fallacy occurs in relation to a problem, let us say, of political economy, the person who has had no logical training may fail to see that there is a fallacy, while the student of logic finds that, though he has had no special education along the lines of political economy, he can as easily detect the flaw in the complicated problem as he could in the comparatively simple generalization about cleverness and mischievousness. Even in matters of everyday experience, the logical mind has the great advantage of a training which renders it as sensitive to an inaccuracy of thought as the ear of an educated person is to a flagrant grammatical blunder. Habits of slipshod reasoning may render the natural thinking faculty dull to the enormity of such an inference as is implied in the statement "I don't believe it, for I saw it in the newspaper, and the newspapers are always telling lies." The mind trained in logical methods is certainly less liable to fall into this and other faults of reasoning, and is correspondingly quick to detect such faults in a written or an oral statement.

Logic and Pedagogy. Logic in its application to the work of the teacher will be given special attention in this course. It need hardly be said, however, that logic is not here presented as a substitute for a knowledge of those subjects which the teacher is called on to teach. There is no method, not even the logical method, by means of which "every one can teach; and, moreover, can teach that which he does not know himself."[1] What is claimed for logic is that, (1) In its purely negative phase, it will enable the teacher both to avoid inaccurate thought himself and to correct it in the pupil, with the additional advantage of enabling him to indicate the source of error. (2) In its positive phase, since pupils are taught as much by imitation as by precept, the logical training which the teacher has acquired will enable him to present in an orderly, consistent, valid, manner any subject which he is qualified to teach, and thus enable him to build up in the minds of the pupils habits of correct thought. (3) Since education does not consist merely in filling the mind of the pupil with knowledge, but has for its chief function "the development of the individual in such wise that life may yield the greatest possible amount of joy and happiness to him and through him to the social group in which he lives,"[2] it follows that (a) the pupil must be put in possession of his social inheritance in such a way as to fit him by general culture to take his place in the social group in which he lives. And this

[1]This was a saying of Jacotot; quoted by Quick, *Educational Reformers*, p. 417.

[2]Shields, *The Psychology of Education*, Lesson XXIV, p. 320.

is not to be done in a haphazard way. Logic analyses the processes by which the race has in the past come to a knowledge and understanding of the universe, and an insight into the nature of those processes, such as logic affords, is a necessary prerequisite to the task of enabling the pupil to rethink those processes correctly. (b) It follows, moreover, that, since the definition just given of the chief function of education contains a reference to joy and happiness, which are ethical concepts, the pupil should be taught to study the relation of facts not only to abstract intellectual principles, but also to definite moral purposes. He should be taught to have clear conceptions of moral values and to adapt his conduct in an intelligent way to the ideals thus acquired. For this, not only strength of character and what is known as moral fiber is required, but also the ability to think clearly, judge consistently, and infer validly. There can be no doubt that the training which logic imparts enables the teacher to cultivate this power in the mind of the pupil.

Applications of Logic. The application of logic to the various sciences, to the theory and practice of law, to the theory and practice of medicine, to the study of history, philosophy, theology, the natural sciences, the sociological sciences, and the philological group requires no elaborate statement at this stage of our study.

It will be made clear by use of examples from each of these branches of knowledge, according as occasion offers in this course of lessons.

History of Logic. A knowledge of the history of logic is not part of a practical course of logic. In order, however, to understand the references which will be given to the various works on logic it will be well to bear in mind the following historical data: (a) The group of Greek thinkers known as the Sophists were the first to bring the art of dialectical reasoning to a high degree of complexity. The Sophists, however, did not analyse the processes of thought for the purpose of determining the rules of reasoning. (b) Socrates, whose entire activity as a teacher was aimed at counteracting the influence of the Sophists, called attention to the need of systematic formation and accurate definition of our ideas, or concepts. To this Plato, his pupil, added his contribution, a set of rules for definition and division of ideas.

(c) Aristotle, by first analysing the processes by which we form our ideas, pronounce judgments, and infer conclusions, and then drawing up a set of rules for the direction of the mind in these processes, founded the science and art of logic.

(d) With the exception of the logical analysis of hypothetical, or conditional, reasoning, which was a contribution of the Stoic School, logic received no substantial addition from the days of Aristotle to those of Francis Bacon. The contributions of Boethius and his followers, the Scholastics, did not change the essential nature of logical teaching, though they included the addition of many improvements in matters of detail.

(e) Francis Bacon (1561-1626), developing hints contained in the works of his namesake, Friar Bacon, shifted the center of logical method from the discussion of principles to the investigation of facts.

We shall learn, later, to style this change a substitution of induction for deduction. The work of Bacon in which this change is advocated is entitled the "Novum Organum."

(f) The famous "Port Royal Logic" (published in 1662) embodied Descartes' reassertion of the Deductive method which Bacon had discarded. With the exception of this, all the great works on Logic in modern times lean towards the Baconian view of logical method. This is notably true of the English writers, Sir John Herschell (1792-1871), Whewell (1794-1866), and John Stuart Mill (1806-1873). In Germany the work of Kant, Hegel and others turned attention to the philosophy of logic.

The Catholic text-books on Logic, including a large number of Latin text-books used in our seminaries and the manuals in English by Fathers Clarke ("Logic," Stonyhurst Series, London and N. Y., 1888), and Joyce ("Principles of Logic," London and N. Y., 1908) adopt the Aristotelian view of logical method and criticise the Baconian theory of Induction from the point of view of the Aristotelian theory of knowledge. John Stuart Mill's "System of Logic" (9 ed. Lond., 1875) is diametrically opposed to the Aristotelian view. A tendency less opposed to Aristotelianism is noticeable in Sir William Hamilton's "Lectures on Logic" (Lond. 1865) and Whately's "Elements of Logic" (London, 1840).

Text-Books on Logic. The following text-books are the most commonly used in the colleges in this country: Clarke, "Logic," Stonyhurst Series, (Lond. and N. Y., 1888); Joyce, "Principles of Logic" (Lond. and N. Y., 1908)—both of these are written from the Catholic point of view in philosophy; Jevons, "Elementary Lessons in Logic" (Lond. and N. Y., 1870);

Bain, "Logic Inductive and Deductive" (N. Y., 1893); Hyslop, "Elements of Logic" (N. Y., 1892); Minto, "Logic Inductive and Deductive" (N. Y., 1894); Creighton, "An Introductory Logic" (N. Y., 1906; new Ed. 1909); Davis, "Elements of Deductive Logic" (N. Y., 1893); Keynes, "Formal Logic" (N. Y., 1894); Welton, "Manual of Logic" (2 vols., Lond., 1904). The last three give special attention to exercises for beginners. Jevons' work is sanely critical of the Mill-Bacon method. Minto's is, perhaps, too philosophical for the beginner. Hyslop's and Creighton's manuals are very much used in normal schools and colleges.

Lafleur's "Illustrations of Logic" (Boston, 1899) is a useful manual for exercise in logical analysis. Welton's "The Logical Basis of Education" (Lond., 1901) does not quite come up to the expectations which its title arouses. It has, however, some good introductory chapters on knowledge and the postulates of knowledge.

CHAPTER II

Mental Images and Terms

REASONING, JUDGMENT, MENTAL IMAGES. The complex mental process expressed by means of the sentences "All colors are physical phenomena. Blue is a color. Therefore, blue is a physical phenomenon," is, as we have seen, the process called *reasoning*. It may, as we have seen also, be divided into three simpler processes expressed by the sentences singly; thus (1) "All colors are physical phenomena"; (2) "Blue is a color"; (3) "Blue is a physical phenomenon." These processes are called *judgments,* and the sentences which express them are called propositions. Further, we may divide each proposition into still simpler elements, represented by one word or several words, thus, "color," "blue," "physical phenomenon." These are called *terms,* and stand for the very simplest of the mental processes studied in logic, or rather, for the results of those processes, namely, *ideas, or mental images.* This taking apart of the process of reasoning, this breaking up of the logical total into its logical elements, should offer no special difficulty. A little practice will enable the student to perform this exercise as easily as he analyses a word into syllables and letters; "transatlantic," for example, into "trans" "at" "lan" "tic," and "trans" into t, r, a, n, s. Of course, the process may be reversed. Just as we can build syllables from letters, and words from syllables, so we can build propositions from terms, and reasoning processes from propositions. From the terms "trees," "plants," "oaks," we can construct the propositions "All trees are plants," "All oaks are trees," and, in the next stage of construction, putting these propositions together and inferring "Therefore all oaks are plants" we have constructed a process of reasoning. The act of inference itself is, indeed, simple and indivisible: in its external expression and structure, however, it has as constituents propositions and terms.

Division of Logic. "Terms," "Propositions," "Reasoning," or (if we go behind the expression to the thought-process expressed) "mental images," "judgments," "reasoning"—these are the three divisions into which logic naturally falls, the three parts, as they are generally called, of logic. We begin with the elements of thought, namely, mental images, and the expression of them in terms; then we proceed to the study of judgments and the expression of them in propositions; and finally we pass to the study of the integral process of reasoning. We study mental images for the purpose of securing *clearness and order;* we study judgments chiefly for the purpose of being able to determine when two judgments are *incompatible, or inconsistent, or opposed, or dependent one on the other;* and we study reasoning for the purpose of being able to judge whether a reasoning process is *valid,* that is to say, *sound,* or *invalid,* that is, *fallacious.*

Idea, Concept, Percept, Mental Image. When, therefore, we take up at this point the study of mental images and terms, we are not concerned with the problem of the origin of our mental images of things, nor with the way in which those images are formed. All logic undertakes to do is to teach us how to arrange our mental images in an orderly manner, and how to define them properly so that they may represent clearly and distinctly, not obscurely and confusedly. Nevertheless, some discussion of the nature and function of mental images must precede the question of clearness and order. What is a mental image? It is what we call an "idea," "a notion," "a concept," "a percept," "an impression." If the English language had a fixed and universally acknowledged terminology in philosophy a great deal of confusion would be saved to the beginner in logic. Unfortunately, it has not. The word "idea," for instance, has a variety of meanings, although, strictly speaking, an idea should mean a highly intellectual mental image, the result, usually, of comparison, reflection, generalization or abstraction. In this strict sense, the word should not be applied to the mental processes of lower animals. It is not correct to speak of a dog's idea or a horse's idea. Similarly, a "concept" is a mental image formed by putting together simple impressions or percepts, and is also a product of intellectual activity, beyond the reach of the mental faculties of the lower animals. "Percept," again, has a restricted meaning, and its relation to "concept" is a matter about which psychologists are concerned. The expression which seems to include them all, "idea," "concept," "percept," etc., is "mental image," which stands related to them as the larger class "tree" stands to the smaller classes "elm," "oak," "pine," etc., so that every idea is a mental image, but not every mental image is an idea.

WHAT IS A MENTAL IMAGE? By a mental image is meant, then, any kind of representation in the mind, so long as it is a mere representation. In other words, it is a picture of the object in the mind. A tree or a house is reflected on the clear surface of the lake. That is an image. If the lake were conscious, or, as we say, could feel the image on its surface, the image would be a mental one. The seal makes an impression on the molten wax; if the wax were conscious of the impression, the impression would be a mental image of the design of the seal. In the case of human beings and the other higher animals, the senses, that is to say, the external senses, sight, hearing, taste, etc., receive impressions in such a way that the person or animal receiving the impression is conscious of it, and is, for that reason, said to have a mental image, picture, or representation. The memory stores up, and the imagination can construct, images, in the absence of the object. Thus I have a mental image, at first hand, of the white page on which I am writing; I can recall the odor of the rose, the taste of an apple, the smell of an orange; I can imagine that the book before me is red, whereas it is brown. These are mental images, representations, or pictures. By comparing, sifting, separating (abstracting) my actual and recalled impressions of various oranges, apples, or roses, I can construct a mental image of "an orange," "an apple," "a rose," or more generally still, "a fruit," "a flower." These are the higher kind of mental images, properly called ideas. But, they are, for our purpose, mental images, merely. A mental image is any representation[3] of an object in the mind, whether it be acquired by one external sense,

[3]In that portion of philosophy which discusses the value of knowledge and is now generally called *Epistemology*, the question is discussed: To what extent is the mental image a true representation of something outside the mind? The question does not belong to logic. In logic it is sufficient for our purpose if we assume that the mental image is a representation, of some kind.

or by several, or by the internal sense (imagination), or by a highly elaborate mental process (ideation).

Various Kinds of Mental Images. Since a mental image is a representation in the mind (subject) of something (object), there are various kinds of mental images according to (A) the way in which it represents, and (B) that which it represents.

(A) Clear, Distinct, Obscure, Confused. A mental image may represent the object clearly ("Clear Idea"), showing the object to us in such a way that we can distinguish that object from others. Thus I have a clear idea of Cologne Cathedral if I can distinguish it from the Cathedral of Milan. Secondly, it represents the object not only with clearness but also with distinctness ("Distinct Idea") when it shows the object to us in such a way that we can not only distinguish it from other objects but indicate also the points of difference. The person, for instance, who has a distinct mental image of Cologne Cathedral can tell wherein it differs from the Cathedral of Milan; he can point out the difference in the size, in the proportions, in the style and size of the towers, etc. The opposite of "clear" is "obscure"; the opposite of "distinct" is "confused."

(B) If we consider what the mental image represents, we have the following kinds of mental images:

(a) *Singular and General, or Universal.* A mental image may represent one person, one object, or one place, such as "The First Emperor of Rome," "Abraham Lincoln," "this desk," "the biggest ship afloat," "Rome," "The Capital of the German Empire," or it may represent several, or even a whole multitude of, persons, objects, places; for instance, "Roman," "American," "desk," "ship," "city," "capital city."

(b) *Concrete and Abstract.* A mental image may represent an object just as it is in nature, a quality or qualities together with the subject, for instance, "this red apple," "a brown house," "a sour green apple," "an honest man." This is called a concrete image. Or it may represent a quality separated, or abstracted, from its subject, as "redness," "brown color," "sourness," "honesty." This is called an abstract mental image.

(c) *Positive, Negative and Privative.* A mental image may represent the presence of something, such as "just," "true," "consistent," "white"; or the absence of something, such as "unjust," "untrue," "inconsistent," "not white." The word, or term, which expresses a negative mental image generally begins with *not-, non-, in-, un-,* or ends with the termination *-less.* This, however, is not always the case. "Darkness" is a negative, since it means merely the absence of light; "ignorance" is a negative, if it means merely the absence of knowledge. On the other hand "immortal" (*in*-mortal) is not a negative mental image, although the word is negative in form; because it represents the presence of a perfection, not the absence of something. So, also, "Infinite," "incorruptible," "immense," etc. A "Privative" mental image is one which represents the absence of some quality or attribute which belongs to the perfection of an object, for instance, "blind," "lame," "sick."

(d) *Collective and Distributive.* A mental image may represent a group as a unit, as when we represent the aggregate existence or action of "a regiment," "a jury," "a baseball team," "a college," "a school"; or it may represent the group as individuals, for instance, "the soldiers in the regiment," "the men of the jury," "the players in a team," "the students of a college," "the pupils of a school." It is only in the judgment or proposition that we can determine whether a mental image is to be taken collectively or distributively. For example, "The jury returned a verdict of guilty," "The jury dined late"; it is clear that in the former case we speak of the jury collectively and in the latter we refer to the individual actions of the jurymen.

TERMS. A "term," as we have seen, is a word, or group of words, expressing a mental image. The definition usually given in textbooks on logic is taken from the work of Thomas Hobbes, the English philosopher (1588-1679). "A term," said Hobbes, "is a word taken at pleasure to serve as a mark which may raise in our mind a thought like to some thought which we had before, and which, being pronounced to others, may be to them a sign of what thought the speaker had before in his mind." This is a correct description of the function of a term, except that it is not strictly true that terms are "taken at pleasure." Because, as far as we are concerned, words have a definite meaning fixed by convention; and, besides, those who first used the words which we now use may have selected them not arbitrarily but in obedience to the laws of phonetics. Hobbes, however, correctly describes a term as a sign of a mental image or, as he says, of a thought. The word "term" itself was chosen because, if we analyse a proposition we find that it is bounded, or terminated, at either end by a word or group of words signifying a mental image. Thus, "Industry is a good quality in a student," "Industry" is one term and "good quality in a student" is the other term of the proposition.

Various Kinds of Terms. It is obvious that, since a term is the expression of a mental image, there are as many classes, or kinds, of terms as there are kinds of mental images given above under the head B. We distinguish, accordingly:

(a) *General and Singular Terms.*
(b) *Concrete and Abstract Terms.*
(c) *Positive, Negative and Privative Terms.*
(d) *Collective and Distributive Terms.*

Besides, there are distinctions of terms which are peculiar to terms themselves, and not paralleled by distinctions of mental images. Thus, a word may be such that, standing for a complete mental image, it may be made the subject or predicate of a sentence, such as "man," "dictionary," etc.; or it may be such that, forming only part of a mental image, it cannot be made the subject or predicate of a sentence, such as "all," "every," "some." The former are called "*Categorematic,*" the latter "*Syn-categorematic*" words. The names are derived from the Greek words κατηγορεῖν, to predicate, and σὺν, with. Only a categorematic word can, generally speaking, be considered a term.

Again, we have "*one-word terms*" and "*terms of many words.*" "Man," "Pharoah," "dictionary," "library," "mountain," are one-word terms; "the tallest man in the village," "the Pharaoh of the Exodus," "Smith's Latin-English dictionary," "the Library of the British Museum," "the highest mountain in the world," are many-worded terms.

Further, it often happens that a word has various meanings, so that when it is used as a term, it is not always used in the same signification. Such terms are called "*Ambiguous*" or "*Equivocal.*" The word "Church," for example, may mean a building, or it may mean the congregation of a particular parish, or it may mean the congregation of all the faithful, or it may mean the clerical profession. "St. Peter's is the largest church in the world," "He is a member of Father Letheby's Church," "The Church is One, Holy, Catholic, Apostolic," "St. Alphonsus gave up the Law for the Church," these are examples of the various meanings of one and the same term. When a term has only one meaning it is said to be "*Univocal.*" All technical terms should be univocal, and most of them are. It is part of the study of logic to learn to recognize ambiguity in the use of a term and thus to be able to detect the fallacious use of it. For instance, if one were to reason thus:

Every good law should be obeyed;
The law of gravitation is a good law;
Therefore, the law of gravitation should be obeyed,

there are at least two ambiguities. "Law" means in one case a moral law, and in the other case a physical law, while "should be obeyed" though it can, properly, have reference only to moral obligation, may, by forcing its meaning, have reference to physical necessity.

To this division of terms into ambiguous and equivocal some add a third class, "*Analogous*" terms, though these are, ultimately ambiguous. An "analogous" term is one which indicates an identity of relation or comparison, among the objects which it signifies, whereas those objects are not identical in reality. Thus we speak of a "blade" of grass and the "blade" of a sword, because, while the two things are really different in every other respect, there is a resemblance in the external appearance. The Pope is the "Head" of the Church, by an analogy with the "head" of the animal organism; because the head directs and controls the conscious movements of the body. Similarly, the various occupations and amusements have furnished analogies for moral, political, spiritual and intellectual things, for example, "the anchor of hope," "the governor of a state," "the spur of ambition," "the wheels of progress," "he gave his adversary (in a dialectic encounter) a knock-out blow." In addition to this tendency, which seems to be universal, to transfer sense-pictures to the supersensuous order, there is in the case of the English language a prolific source of analogy and ambiguity in the condition of the historical development of the language itself; the word "mean," for instance, derived from the German "gemein" is identical in form with the word derived through the French "moyen" from the Latin "medium." But between the phrase "mean man" and the phrase "mean distance" there is no identity or univocation, but only a remote analogy, amounting almost to ambiguity.

Logicians also distinguish between *"Relative"* and *"Absolute"* terms. A "Relative Term" is one which in its meaning implies another object. This other object also receives a name (the correlative) from the fact or series of facts which gave the object of the first term its name. Thus "ruler" and "subject," "teacher" and "pupil," "cause" and "effect" are relative terms, and in each pair one is said to be the "correlative" of the other. An absolute term is one which in its meaning implies no reference of this kind to any other object. Of course, nothing finite is absolute in every respect, since it must be at least dependent on something else. Logically, however, the term is absolute when it implies no correlative term.

Finally, the ancient writers on logic discussed the question of *"non-significant"* terms; they inquired, namely, whether there are any terms which have absolutely no meaning. The question has reference to meaning generally, and cannot be answered until we have discussed the extension and comprehension of terms, to which the next chapter will be devoted.

In studying a word in logic we should, if we know its meaning, be able to answer the following questions:

(1) Is it a *categorematic* or a *syncategorematic* word?

(2) Is it a *one-worded* or a *many-worded* term?

(3) Is it a *univocal, equivocal,* or *analogous* term?

(4) Is it a *singular* or a *general* term?

(5) Is it a *concrete* or an *abstract* term?

(6) Is it a *positive, privative,* or *negative* term?

(7) Is it a *collective* or a *distributive* term?

(8) Is it a *relative* or an *absolute* term?

CHAPTER III

The Extension and Comprehension of Terms

Extension and Comprehension. There are two ways of viewing the meaning of any term, two directions, so to speak, in which that meaning reaches out. The term may be viewed in its *Extension* or in its *Comprehension,* that is, it extends in one direction over a number of individuals or groups of objects, and it reaches down in another direction so as to include a number of qualities, attributes, or characteristics. If we take, for example, the term "tree," we shall find, as soon as we begin to reflect on the meaning of it, that it "extends" over a large number of groups of objects, "pines," "oaks," "elms," etc., while if viewed in another aspect, it "comprehends" a large number of attributes or qualities, such as, "substance," a "body," a "living body," a "plant," "perennial," "larger than a shrub." So that, if one is asked the *meaning* of the term "tree" one may answer either (1) by saying "trees" means oaks, pines, elms, etc., or (2) by saying "tree" means a living bodily substance, a perennial plant larger than a shrub. We shall learn later that the second answer is an attempt to give a logical definition of a tree. The two dimensions, as we may call them, may be represented graphically thus:

Tree, elms, oaks, pines, etc.
substance,
body,
living body,
plant,
etc.

The horizontal line represents the extension of the term, and the vertical column the comprehension.

SYNONYMS FOR EXTENSION AND COMPREHENSION. In this instance logic has the disadvantage of being excessively rich in terminology. Hardly any two authors have the same names for Extension and Comprehension. The following is a list of the principal words used by different authors. The words in each column are synonymous:

Extension	Comprehension
Extent	Intent
Denotation	Intension[1]
Sphere	Connotation
Breadth	Depth
Application	Implication
Scope	Force

[1]*Intension* and *Extension* would seem to be preferable to *Comprehension* and *Extension.* In practice, however, it is found that students are very apt to mistake *Intension* for *Intention.*

Each of these has its own particular flavor, so to speak, and may aid us in understanding what is meant by "Extension" and "Comprehension." "Extension" and "Comprehension" seem to us to be the most desirable terms to use, with "Denotation" and "Connotation" as second choice.

Extension Defined. Extension may be defined as "the individuals or groups of individuals included under the term and to which the term may be applied." The extension of the term "Ship" is set forth in the mention of steam-ships and sailing-ships, or yacht, schooner, sloop, brig, etc. The extension of the term "flower" is set forth in the mention of "roses," "violets," "carnations," etc. When we use the term to signify these smaller groups of objects, when we think of these, and "mean" them, then we are using the term in its extension. Sometimes we use it in all its extension, and sometimes in only part of its extension, as when we say "All flowers are plants," "Some flowers are perennial."

Comprehension Defined. Comprehension may be defined as "the attributes, qualities, notes, or characteristics which the term implies. and which must be present in an object before the term can be applied to it." The word "brig," for instance, means a ship with two masts, square-rigged. If, when I use the word, I mean these qualities, or any of them, I am using the term in its comprehension, that is for those attributes which must be present (first the general attributes of a ship, and second, the special attributes of a brig) before an object is called by that name. Besides, the name implies these attributes. They must be present, otherwise the name is misapplied.

WHICH MEANING IS UPPERMOST IN THE MIND? If we stop to think what we mean, precisely, by the words which we use in ordinary conversation we shall discover that we sometimes use terms in extension and sometimes in comprehension. When we say "The Chinese are industrious," we are usually thinking of the class Chinamen (extension), not of the attributes (comprehension) which distinguish a Chinaman from other men; but when we use the word "industrious" we are thinking of an attribute (comprehension) and not, usually, of the class (extension) industrious persons. This means that in the one case the extension, and in the other case the comprehension, is uppermost in our minds; for when we wish to examine the matter more deeply we find that each term has both extension and comprehension. When we say that "All that glitters is not gold," which, as we know, means "Some things that glitter are not gold," we are thinking of the attribute "glitter," and may think either of the class of things called gold, or of the attributes which constitute the comprehension of that class.

RELATION BETWEEN EXTENSION AND COMPREHENSION. Logicians lay down a rule regarding the relation between the extension and the comprehension of a term, though they are not all agreed as to the wording of the rule. We may safely say that "the larger the extension of a term, the smaller its comprehension, and the larger the comprehension, the smaller the extension." If I take the term "ship" and add successively the attributes implied in "steam ship," "war ship" (meaning a modern war ship), "cruiser," "U. S. cruiser," "First Class U. S. Cruiser," "The First Class U. S. Cruiser Brooklyn," I add in each case to the comprehension, but I diminish the extension from the thousands of objects denoted by the first term down to the one object to which the last term is applied.

This truth may be represented graphically thus:

COMPREHENSION:	EXTENSION:
Body	*Minerals, plants, animals, men*
Body with life	*Plants, animals, men*
Body with life and sensation	*Animals, men*
Body with life, sensation, and reason	*Men*

Reading horizontally, we find that the term which has the least comprehension, namely, "body," has the greatest extension. When we add to the comprehension of "body" and make the term to comprehend "body with life," the whole class "minerals" drops out and we have a smaller extension, namely, "plants, animals, men." Next, we add "sensation" to the comprehension, with the result that "plants" drop out of the extension and there are only "animals and men" left. Finally, when, by the addition of "reason" we further increase the comprehension, the class "animals" is excluded and the extension is reduced to the class "men."

It is not accurate to say that the comprehension and extension of terms are in inverse ratio, because sometimes the addition of one attribute diminishes the extension by many thousands, while sometimes the addition of one attribute diminishes the extension by only a few individuals. When, for instance, to the term "Catholic" I add the attribute "French," I diminish the extension of the term by excluding the millions of Catholics who are German, English, Irish, etc., while if to the same term "Catholic" I add the attribute "lay" I diminish the term by the exclusion of all those who are clerics or religious, a large class, but still not so large as the class including German, English, Irish, etc., Catholics, both lay and clerical. The phrase "inverse ratio" implies exact mathematical relation between increased comprehension and diminished extension; and this is not always the case. Nevertheless, it is true in a general way that when the comprehension is increased the extension is diminished and when the extension is increased the comprehension is diminished. The term, therefore, which has the widest extension, namely "Being," is the term which has the least comprehension; and the term which has the greatest comprehension, namely, a singular term, such as "the first Emperor of Rome," has the least extension. "Being," "substance," "body," "living body," "animal," "man," "Roman," "Roman Emperor," "the first Roman Emperor," represents a series descending from the greatest extension to the least, and at the same time it represents a series ascending from the least comprehension to the greatest.

Connotative and Non-Connotative Terms. At the end of Chapter II the question was asked "Are there any terms that are non-significant," that is, terms which have no meaning? The occasion for asking this question arises from the distinction which logicians make between "Connotative" and "Non-Connotative" terms. A Connotative term is one which, while it denotes an individual, or individuals, also signifies, or con-notes, an attribute or several attributes. Thus, the term "tree," as we have seen, denotes elms, pines, oaks, etc., and connotes life, plant organization, size greater than that of a shrub, etc. A non-connotative term is one which merely denotes or designates an attribute, an individual, or several individuals, without connoting, or implying any attribute whatsoever. Our question, then, may be restated in this way: "Are there any non-connotative terms?"

All logicians seem to be agreed on certain points:

(1) *All concrete general terms are connotative,* that is, they denote, or designate, objects and connote or imply attributes. For example, "tree," "horse," "ship," "book," "desk," "flower," designate individual things, or groups of things, to which we apply these terms, and to which alone we must apply them if we use the English language correctly; at the same time, they connote, or imply, certain attributes. We are not allowed to call a horse by the general name "book," because "book" implies certain attributes which a horse has not. On this point there is no controversy.

(2) *Some abstract names are connotative.* There is no reason why a term which denotes an abstract quality may not connote, or imply, other qualities. For example, our experience of "sweetness" is associated, as a general rule, with "pleasantness"; the term "sweetness" may, therefore, be said to connote "pleasant-

ness" at the same time as it denotes the quality sweetness. On this point most logicians agree.

(3) *Singular names may be, and* (when they are not proper names) *generally are, connotative.* Thus, "The present Pope," the (singular) name of one person, not only designates Pius X, but also implies very distinctly his august office as head of the Church. "The highest mountain in Europe" not only designates, or denotes, Mt. Blanc, but also connotes the qualities comprehended by the term "mountain" and also the additional attribute "highest." Here, once more, there is little room for controversy.

(4) *When, however, we come to proper names,* such as "Pius X," "Mt. Blanc," "Rome," "New York," "James Smith," "George Washington," etc., logicians are not all agreed as to whether these terms are purely denotative, that is, non-connotative, or whether they are, on the contrary, truly connotative. English authors especially, are divided on this point, owing to the divergence of opinion between Mill, who held that proper names are non-connotative, and Jevons, who advocated the opposite. It will be best to distinguish various points into which the problem naturally resolves itself:

(a) *Proper names may have been originally bestowed on account of some attribute or* *quality*. This is true of place names and of personal names. "Missouri" is a name derived from the color of a river, "Teutopolis," "Kilkenny," "Little Canada," "New Prague," are names which refer to the racial character of their first settlers. "Smith" is evidently a trade name, "Weidenborner" is just as clearly a name originally given to one who held property near some spring where willows abounded. But, the name once given for some specific reason may cease to have any reference to that reason. The descendants of the Smith may be lawyers or farmers, or clergymen, the river that gives its name to a state may change its color and the name will, nevertheless, be used to designate the state.

(b) *Some proper names appear to have a connotation.* John Smith is a so-called Anglo-Saxon name, Giuseppe Garibaldi is distinctively Italian, as Bismarck is distinctively German. But, it is evident that these names do not connote nationality; an Indian may be called John Smith, and an Anglo-Saxon may be called Garibaldi or Bismarck if his parents decided to honor in that way the memory of their favorite political heroes. "George Eliot," a distinctively masculine name, is used to designate a woman.

(c) *We must carefully distinguish between a semblance of connotation,* that is, the information which we associate with the name of a person or a place known to us, *and the true connotation,* that is, the meaning implied by the name. Thus one naturally associates with the name of an acquaintance or friend, John Thompson, his appearance, his character, his occupation, etc. This information, however, is not implied by the name. John Thompson may have his name changed by act of legislature and thus dissociate it forever from the information with which we had associated it. But if John Thompson were to try to change the meaning of a common name which we apply to him, e. g., "lawyer," "Democrat," "tall man," "honest man," he would find that he could not, because these terms imply certain attributes.

Between the information which we associate with a proper name and the proper name itself there is a purely accidental bond; between the connoted attributes and the name which connotes them there is a bond which is founded on a universal convention among those who use the language. This may be illustrated by supposing that a boy has a dog which he names "Dewey" and a pony which he names "Bob." He may change the names at any time and call the dog "Bob" and the pony "Dewey." But he may not call the pony a dog nor the dog a pony.

PROPER NAMES ARE NON-CONNOTATIVE. *In this* *restricted use* of the word "Connotative," when, namely, we understand the term to mean that certain attributes are necessarily implied by the term, Proper Names are not connotative, and, in that sense, since they merely denote objects, they are non-significant. Of course, when, as often happens, the proper name is used to designate a class, "A Daniel come to Judgment," "a Nero," "a Solomon," "a Napoleon," it is no longer a proper name but a general term, and is connotative.

Have Proper Names Comprehension? Sometimes Comprehension is taken in its strictest sense, as synonymous with connotation; sometimes, however, its meaning is broadened so as to include not only the attributes and qualities implied by the term, but also those attributes or qualities which, for any reason, we associate with the term. In this wider sense of the word, Comprehension is, obviously, applicable to Proper Names. Returning, now, to the question of abstract names, treated above under No. 2, we see that it is only in this larger use of the word that abstract terms have comprehension. Properly speaking, they have no connotation.

Extensive and Comprehensive Meaning of Terms. Every term has, therefore, a two-fold meaning, extensive and comprehensive. In some cases, namely, in the case of concrete general names, the two are equally important.

Psychologically, one may be more natural than the other, in so far as it is more prominent in consciousness. In the case of abstract terms and proper names, there is comprehension in the larger sense of the word, but not in the narrower sense, in which comprehension is made to mean that which is necessarily implied in the word as we ordinarily use it.

EMOTIONAL VALUE OF NAMES. The emotional value of names has not been touched upon here, as being foreign to the logical problem before us. Names have, of course, an emotional value. The general names in our own language are for us fuller of emotional content than their equivalents in any other language. And proper names, it need hardly be said, have a still richer emotional content. It would be absurd to say that the names of our relatives and friends, the names of our national heroes, and the names of Christ, Our Blessed Lady and the Saints, which every Christian regards as sacred, have no "meaning." The illustration serves to show what a vast difference there is between logic and life. Logic treats of names merely to determine their extension, comprehension and connotation, while for the purposes of life the emotional meaning and "force" of names must also be considered. Logic merely aims at teaching us to define and divide terms accurately and so attain clearness and distinctness in our mental images. Thus, it prepares the way for consistent judging and valid reasoning. For the purposes of life, words are to be considered in their emotional as well as in their purely logical aspect. When clearness, distinctness, consistency and validity of thinking are brought to bear on right action and rich vital expression, logic is applied to life and made efficient in the highest sense.

CHAPTER IV

Definition

THE PURPOSE OF DEFINITION. There was a time when precision of ideas was attained merely by comparison, reference to examples, the citation of instances, etc. When Socrates appeared a teacher at Athens, he found that his contemporaries, when asked "What is courage?" would answer by saying "Courage is to the soldier what wisdom is to the ruler," or by referring to Achilles as a type of a courageous man, or by saying that one who singlehanded could hold a pass against ten oncoming enemies, was courageous. The reform which Socrates wrought in the study of human conduct consisted in substituting for these extrinsic and inadequate methods the method of determining what courage, or any other concept, is by pointing out the attributes, notes, or characteristics which constitute it. That is to say, he introduced Definition as the only scientific method of securing clearness and distinctness in our mental images and accuracy in our use of terms.

WHAT IS DEFINITION? Definition, in general, is the analysis of the comprehension of a term, or the setting forth in explicit statement those attributes, qualities or characteristics which are comprehended necessarily in a term, and which necessarily belong to the thing which the term signifies.

NOMINAL AND REAL DEFINITION. A Definition is either "Nominal" or "Real." Sometimes we use a definition in order to set forth the etymological force of a word, as when we say that logic is derived from the Greek word λόγος, meaning "reason" or "discourse," or when we say that a Cathedral is so called because in it the bishop has his chair, which in Greek is called *καθέδρα*. Sometimes we give the meaning which a word has in common, everyday use, as when we explain "van" as the British equivalent of our word "baggage-car." Again, we may give the technical use of a word, as when we say that "Bellis perennis" is the botanical name for a "daisy." In all these instances we are dealing with words only; the definition is, therefore, Nominal, that is, *either etymological, usual,* or *technical.* In a *Real* definition we attempt to define what the object is which is designated by the term: As "Logic is the science and art which so directs the mind, etc.," or "van" is the part of the train used for carrying baggage, or, "a daisy is a plant of the genus bellis, etc."

DESCRIPTION AND LOGICAL DEFINITION. Of real definitions there are two distinct kinds, the one commonly called "Description," and the other "Essential Definition," or Definition in the strictest sense. A description is, of its nature, inadequate from the point of view of logic—it either characterizes the object by some accidental qualities which serve, indeed, to identify it, but do not refer to its intrinsic nature, or in some other way a description deals with what is extrinsic to the thing to be defined. There are various kinds of description: (a) *Description by accidental qualities;* for instance, "Man is an animal that cooks his food." This does not define "man" adequately. For while it is true that man is the only animal that cooks his food, this characteristic does not belong to the intrinsic nature of man; the description does not set forth adequately the connotation of the term "man." (b) *Description by examples,* for instance, "What is a Science? Botany, Geology, Logic, Mathematics are Sciences." This is a very superficial answer to the question, because it does not go into the comparison of the various sciences for the purpose of determining analytically what makes them to be sciences, and what "constitutes" a science. (c) *Description by reference to types;* for instance, if when asked "What is a heroine?" we answer "Joan of Arc," "What is a tyrant?" we answer "Nero," we give a concrete embodiment of the qualities which constitute a heroine, or a tyrant, but, as in *b*, we do not analyse those qualities. (d) *Description by reference to use;* for instance, one may describe the Roman "stilus" as the instrument used by the Romans in writing. This, however, is not even an adequate description, because one would expect a reference to the material, size, shape, etc., of the stilus. And even when those elements are included, there should be an orderly or logical arrangement of them; otherwise, the description is a description merely, and not a definition. (e) Finally there is *description by extension,* as when one answers the question, "What is a tree?" by saying "Oaks, elms, firs, etc., are trees." Here there is analysis, but it is analysis of the extension, not of the comprehension, and the result is a division, not a definition.

WHAT IS ESSENTIAL OR LOGICAL DEFINITION? The "Real Definition," in the strictest sense, or the "Essential Definition," as it is sometimes called, analyses the comprehension of the term and sets forth, as the result of that analysis, those attributes or qualities which are intrinsic to the object defined. Thus definition enables us not only to distinguish the object defined from other objects, but also to tell wherein it differs from them. For the aim of all logical definition is to make our ideas distinct as well as clear.

DEFINITION IMPLIES ANALYSIS. Analysis, or a certain amount of analysis, according to the nature of the thing to be defined, must go before the act of formulating our definition. This means that we must reflect on the nature of the thing to be defined, and by reflection, comparison, and sifting of our ideas about it, determine what is part of the intrinsic nature of the thing, and what is accidental. This analysis is, as the name implies, a separation. It is very much like the process of physical separation by which a mechanic, for example, takes a clock to pieces in order to study its mechanism. In the case of the logical analysis, however, we resolve the total into "logical" parts, not into "physical" parts. For instance, when we study a triangle we do not take the physical parts of it asunder, but we consider the logical constituents of it, that is, we consider the attributes, (1) "rectilineal figure," (2) "three sides," (3) "three angles," (4) "sum of all three angles equal to two right angles." Next, we compare and sift these attributes, and find that 3 and 4 follow from 2, are, in a sense, included in it. We find also that "rectilineal figure" is an attribute common to many other mathematical objects, such as "parallelogram," "square," etc., but that "three-sided" is peculiar to triangle. We have here, then, all the essential constituents of the thing defined, and the result of our analysis is the definition, "A triangle is a rectilineal figure of three sides."

Requisites of a Definition are Exactness, Clearness, Brevity. If we analyse the comprehension of a term, setting aside those attributes which are accidental, and excluding those which are inferences from others, we have left the attributes which are intrinsic to the thing defined, which make it to be what it is. If we do this, our definition will be exact. If we remember, moreover, that the purpose of definition is to insure clearness and precision in our ideas, and that in any sentence, especially in a sentence used in scientific writing, unnecessary words are a source of confusion, we realize that all the conditions of a good definition are summed up in the three words "exact," "clear," "brief."

Rule I. A Definition Should Be Exact. It should apply to all the thing defined and only to the thing defined. Like a well fitting garment, it should be neither too narrow nor too wide. "A horse is a domestic animal" is too wide a definition. "A horse is a domestic animal that is used for drawing wagons" is too narrow. "Wide" and "narrow" here apply to the extension of the term. In reality, since the former definition does not include enough attributes, it is too narrow in comprehension, for it does not include the qualities by which horses are distinguished from cows and other domestic animals. On the contrary, the latter definition includes too much in comprehension, for, that a horse is "used in drawing a wagon" is not part of the essential comprehension of the term "horse." If the definition includes only the essential qualities of the thing defined, and includes all of them, it must be exact, it can be neither too wide nor too narrow.

This is the reason why the first rule of definition is often formulated as follows: *"A definition should consist of the genus proximum and differentia ultima."* By *genus proximum* is meant the nearest group or class to which the object belongs, and by the *differentia ultima* is meant the distinguishing feature, or character, by which the object is marked off from the other objects of the same proximate generic class. The meaning of the rule thus technically formulated is the same as that of our first statement, "A definition should be exact." For instance:

TERM:	PROXIMATE GENUS:	DIFFERENCE:
Triangle	Figure	Three-sided
Man	Animal	Rational

The part of the comprehension which is common to circles, squares, etc., namely, "figure," and the part which is peculiar to triangles, namely, "three-sided," when added, constitute the whole of the comprehension of "triangle." Similarly, in all other cases, a definition by proximate genus and ultimate difference is certain to be exact, because it includes all the comprehension.

Rule II. A Definition Should Be Clear. More specifically, it should be clearer than the thing defined. For it is, as we have seen, the purpose of a definition to enable us to have clear ideas. This rule includes several subordinate rules:

(a) A definition *should not be a case of "ignotum per ignotius"* (the unknown by the more unknown). Thus, Dr. Johnson's definition of a "net" as "a reticulated fabric decussated at regular intervals" is not likely to make the concept of a net clearer to the ordinary mind. The same fault must be found with definitions such as "Fluency is the exuberance of verbosity." However, an exception must be made in favor of the scientist, who has a right to define a scientific term in the phraseology of his science, although by so doing he renders the idea more obscure for the ordinary non-technical person. Thus, it would not enlighten the average inquirer concerning the nature of the soul to be told "The soul, my dear sir, is the first entelechy of the organized body having the potency of life"; yet, to an Aristotelian metaphysician the sentence would be perfectly intelligible, and, no doubt, would by him be accepted as a clear definition.

(b) *It should not be "idem per idem,"* that is, the definition should not contain the word defined or a derivative from it. It does not, for example, clear up our concept of "metal" to be told that it is "a metallic substance." It is not a good definition of "liberty" to say that it is the condition of "freedom," for one is merely the Latin and the other the Anglo-Saxon word for the same thing.

(c) *It should be expressed in literal, not in figurative, language.* "Logic is the medicine of the mind," "Sleep is the brother of death," "The sea is the image of eternity"—these and similar figurative expressions appeal to the imagination and the feelings, but they do not bring us any nearer to a logical analysis of logic, sleep, the sea.

(d) *It should, if possible, be expressed in affirmative, not in negative, terms.* We get a much clearer notion of a thing when we are told what it is than when we learn merely what it is not. The number of things which a thing is not is infinite, the number of things which it is, is necessarily restricted.

RULE III. A DEFINITION SHOULD BE BRIEF. Conciseness is nowhere more necessary than in the framing of definitions. Elsewhere, redundancy may have its occasional uses. Here, however, the strictest care must be exercised not to include words which are unnecessary. And in this context it may be remarked that certain other stylistic requirements should be insisted on in framing definitions. The most common fault of construction in expressing definitions is illustrated by the following: "Piety is when one says one's prayers with devotion, etc."; "Honesty is when one doesn't steal, etc." These definitions should be cast in the form, "Piety is a quality, etc."; "Honesty is a quality, etc."

LIMITS TO OUR POWER OF DEFINING. There are limits to our power of defining. Some terms resist all our attempts to define them and others are very difficult to define. The term "God" cannot be defined, because no creature can adequately comprehend its meaning. "Being" cannot be defined, because its comprehension, since it is the minimum, cannot be divided into component elements. Primary feelings and simple sensations, such as "pain," "thirst," "whiteness," cannot be defined, because there are no simpler elements of feeling or sensation into which they can be resolved, though, of course, they can be described, and the physical or physiological conditions which accompany them or cause them can be determined. Many of the objects of our everyday experience are difficult to define, because we think of them in extension and not in comprehension. Every child knows what a dog or a cat is; yet few can define the terms "dog" and "cat," because few have thought of the qualities which are included in the comprehension of those terms, although nearly all know the extension, that is, the objects and classes of objects to which the terms are applied. An effort should, however, be made to enlarge the number of terms that we can define; because the process of analysing the comprehension of the terms which we use is an excellent training, and, besides, a term once defined is clearer and more distinct in our consciousness, and is less liable to be used inaccurately.

*   *   *   *   *

Logic of Description. There is, properly, no logic of Description. The art of describing belongs to rhetoric rather than to logic. Still, between the rules of definition and the rules of description there is a resemblance. (a) A definition should be exact—corresponding to this is the rule of description according to which *a description should be true.* It is not necessary to reproduce all the details in the description of a scene, a person, an event. But the details selected should, so to speak, be axial; that is, they should be such that the other attributes can be grouped around them in such a manner as to form a consistent picture. The novelist who specified the afternoon as the time of a military function and then sent the whole company of knights and ladies from that function straight to Mass in the royal chapel overlooked, through ignorance, the rule of which we are speaking. (b) A definition should be clear—*a description should be vivid.* The purpose of the description will, of course, determine the kind of vividness that is more desirable. Sounds, colors, outlines, impressions, moods may be accentuated according to the aim of the person who describes the scene or event. (c) A definition should be brief—*a description does not demand the same rigorous brevity as the definition does.* Yet it is true of a description as it is of a definition that each word should serve some useful purpose, either visual representation, auditive representation or emotional association.

Description by Use. When we describe a thing by its use we should *refer to its primary use.* A chair would not be correctly described by saying "it is a piece of furniture on which we stand when we hang pictures." In a recent work of fiction a greenhouse is incorrectly described as "the place where the garden sprinkler is kept when it isn't in use."

Description by Accidental Qualities. Again when we *describe an object by its accidental qualities* we should not select those qualities at random or arbitrarily. Dr. Johnson, when he described oatmeal as "the food of Scotchmen," selected an entirely extrinsic characteristic of the thing described.

Description by Type or Example. Finally, in describing things by type or example we should be sure to select prominent types and striking examples, so that the concrete instance which we advance shall be clearer than the general, or abstract, quality which it illustrates.

It should also be remembered that a description differs from a definition in this, that while the latter aims at precision or limitation of one's ideas, the former may be intended merely to suggest and, consequently, aims at putting our ideas in relation to other ideas.

CHAPTER V

Division

What is Logical Division? Logical Division is related to the extension of a term in the same way as Definition is related to the comprehension. Just as in Definition we analyse and arrange the comprehension of a term, so in Division we analyse and arrange the extension of a term. In other words, Division takes a large group, or class, of objects, breaks it up into smaller groups and classes, and arranges these in an orderly manner.

Purpose of Division. The purpose of Division is similar to that of Definition. If we are to have clearness and precision in our mental images of things and accuracy in the use of words, it is necessary not only to define, that is, to determine the limits of the comprehension of a term, but also to arrange the extension of the term in such a way that we can hold in our minds in an orderly manner the various groups of objects to which the term is applied and the relation of those groups to one another. For the purpose of study, too, it is always useful, and sometimes necessary, to divide the subject of our study in such a way as to proceed from one department, or branch, of the subject to other departments, or branches, in an orderly manner. If order is "Heaven's first law" in the physical universe, it is no less truly an important law of mind. This much, at least, order will ensure us, the avoidance of unnecessary repetition and the removal of those difficulties which arise from disorder, confusion, and mental slovenliness. The ease, the sureness, the celerity with which a regiment can be transferred from one place to another, compared with the difficulty, uncertainty, the slowness of the transfer of an unorganized mob of men is an image of the advantage which the mind can derive from an orderly marshalling of its ideas under appropriate Heads of Division and Sub-Division.

Logical Division and Physical Partition. It is necessary to recall here a distinction to which reference was made in the preceding chapter. A Logical Division differs from a Physical Division, or Partition. If we study the object denoted by the term "ship," we perceive at once that that object consists of various parts, hull, deck, mast, rigging, etc. To take these asunder would be a process of physical partition. We perceive also that the "class" ship includes various subordinate classes, steam ship, sailing ship, war ship, brig, schooner, yacht, etc. The process by which we distinguish these various "sub-classes," and arrange them in the natural order of their co-ordination and subordination is logical division.

TECHNICAL TERMS USED. The larger "class" which we break up into the smaller groups or "sub-classes" is called the "*Total Divided,*" the smaller groups are called the "*Dividing Members*" of the division. There is always some "*Principle of Division,*" that is, some quality, character, or distinguishing characteristic, which we select as the basis of our division. When we divide Europeans into Catholics, Anglicans, Lutherans, Presbyterians, it is evident that the Principle of Division is creed. When we divide Europeans into Frenchmen, Italians, Germans, Russians, etc., the Principle of Division is nationality. The Principle of Division may be some inherent characteristic of the object itself, or some use or other external relation of the object. For example, animals may be divided according to structure, or according to their value as human food. The former is an intrinsic, the latter is an extrinsic Principle of Division. Whether we select an intrinsic or an extrinsic Principle of Division will depend on the purpose which we have in mind in making the Division. It may be said here, however, that for scientific purposes it is in most cases advisable to adopt an intrinsic Principle of Division. We shall return to this point under Rule IV.

Requirements of a Logical Division. The general logical purpose of Division, as we have seen, is to secure order, precision, and clearness in our use of terms in extension. This purpose determines the requirements to which all Divisions must conform if they are to be considered logical. The requirements are summed up in the following Rules:

Rule I. Each Division Must Have Only One Principle of Division. To start to break up a large group into smaller groups according to one Principle of Division and then to change to another Principle and complete the Division according to that other Principle can lead to nothing but confusion and disorder. Thus, if we take the class "books" and divide it into folios, quartos, books of poetry, useful books, interesting books, we have four different Principles of Division, and the result is an arrangement which is illogical. A consequence of not adhering to the same Principle of Division is the fact which may be readily observed in the instance given, that *the Dividing Members, or subordinate classes, are not mutually exclusive.* A book may be at once a quarto, a book of poetry, a useful book, and an interesting book. This is always a consequence of failing to adhere to one Principle of Division. For this reason some authors state the first Rule as follows: *The Dividing Members of a Division must be* *mutually exclusive.* In the following general scheme of Division,

$$\mathrm{A}\begin{cases} a & \begin{cases} a^1 \\ a^2 \\ a^3 \end{cases} \\ b \\ c \\ d \end{cases}$$

if A stands for the Total Divided, a, b, c, d, for the Dividing Members, we must be able to say No a is b, No b is c, No c is d; and in the subdivision of a we must be able to affirm No a^1 is a^2, No a^2 is a^3, etc.

An expedient by which we are certain always to have but one Principle of Division and which ensures the mutual exclusiveness of the Dividing Members is that known as "*Dichotomy*" (the name is derived from two Greek words meaning "to cut in two"), which consists in dividing a class A into B and *not-B*, or, in general, into a sub-class and the contradictory of that sub-class. Thus trees are divided into "deciduous" and "non-deciduous" according as they do or do not shed their leaves in autumn, substances are divided into "living" and "not living" according as they do or do not possess life. This, as has been said, possesses the advantage of securing in the strictest way the mutual exclusiveness of the subordinate classes.

On the other hand, it has certain disadvantages. In some cases it is a purely formal process, with no relation to the real properties of things, as when we divide "metals" into "learned" and "not learned," learning being a property which cannot in any real sense be predicated of metals. Besides, even in cases where this method can be applied without creating unreal classes, such as "learned metals," it has the disadvantage of not suggesting any further subdivision.

Rule II. When a Division is completed, the sum of the Dividing Members should be equal to the extension of the Total Divided: $a+b+c+d$ must be equal to A. Thus, if one were to divide "chairs" into "useful" and "ornamental," the division would be defective because (not reckoning the possibility of a chair being both useful and ornamental, which, under Rule I, would condemn the division) there are chairs which are neither useful nor ornamental, and consequently we cannot write "useful chairs + ornamental chairs=chairs." Again, if one were to divide "Roses" into "American Beauties" and "Killarneys," the division would be defective, because the sum of the two sub-classes does not equal the extension of Roses, since it does not include the many other varieties, such as "Bride Roses," "Kaiserins," etc. The implication in a Division is that, if A is divided into *a, b, c,* and *d,* then it must be true that any one object of the class A is either *a* or *b* or *c* or *d.*

Rule III. A Division should be properly co-ordinated and subordinated, or as the old Logicians expressed it, A Division should not make jumps, "*Divisio ne fiat per saltum.*" This means that a large group, the Total Divided, must first be divided into the groups which are nearest it in extension, then into the proximate groups, and finally, into the smallest groups. It is a fault in the arrangement if we jump at once from the largest to some of the smallest groups. Thus, if one were to divide Europeans into Russians, Tuscans, Parisians, etc., one would pass over the classes co-ordinate with Russians, namely, Frenchmen, Germans, Italians, and come down by a jump to the subordinate class Tuscans which means the inhabitants of one province of Italy, and then, by another jump, passing over the classes co-ordinate with Tuscans, namely, Bretons, inhabitants of Normandy, etc., come to the subordinate class Parisians. We may, if we will, imagine that the various classes into which the Total is divided are sticklers for rank and precedence. Those nearest the Total in extension come first in rank, then those farther away, and finally the very small classes which are almost plebeian in their want of rank. Now when we arrange these groups in a logical division, we must, like a hostess at some important social function, be sure to give to each its proper rank. Let us suppose we are asked to divide the class "Americans." Our arrangement should be represented thus:

INHABITANTS OF AMERICA

South Americans North Americans Central Americans

Mexicans Inhab. of U. S. Canadians

Inhab. of Ohio Inhab. of Mass. etc.

Bostonians Citizens of Lowell etc.

If we placed Bostonians in the same rank as Canadians we should, from the point of view of logic, be placing as co-ordinate with Canadians a class which is of subordinate rank. The true co-ordinate of Canadians is inhabitants of the United States, and the true co-ordinate of Bostonians is inhabitants of Quebec. The great advantage in an orderly arrangement in which classes are properly co-ordinated and subordinated is this: in such an arrangement the distance of one group from another is an index of the difference between them. Thus, in a scheme of division of "Living things" the maximum difference in structure will be between the lowest of the "Protophytes" and the highest of the "Metazoa," namely, "Man," while "Man" will be similar in structure to the other "Primates," less similar to "Birds" and still less similar to "Crustaceans," etc.

Rule IV. A Division Should be Based on an Important Principle of Division. The importance of any one quality, or characteristic, as a Principle of Division will depend, as has been said, on the purpose of the Division. If I wish to divide a thousand volumes for the purpose of arranging them on the shelves in such a way as to produce the most pleasing effect, I shall select as the Principle of Division the style, quality, and color of the bindings. If, on the contrary, I wish to arrange the volumes so as to facilitate the finding of a book on a certain subject, I shall select as the Principle of Division the contents of the volumes, Theology, Philosophy, History, Literature, etc. Again, if I am interested in extracting the perfumes of flowers, I shall divide flowers, not according to their structure, but according to the kind of perfume which characterizes each kind. Absolutely speaking, however, a division of books according to content and a division of flowers according to structure is considered to be based on important differences, while a division of books according to the style of binding and a division of flowers according to perfume would be regarded as divisions based on unimportant differences. The reason is that, for general purposes, the content of books or the structure of flowers is more an object of interest than binding and perfume respectively. The Division based on an important Principle is sometimes called *Natural,* while the Division based on unimportant differences is called *Artificial.* It is, of course, only in the case of the Natural Division that the distance between two groups is an index of the difference between them, as was pointed out in Rule III.

LIMITS TO DIVISION. As there are limits to Definition, there are limits also to Division. The Division of the largest group into the next largest is followed by the division of these into still smaller groups, but, finally we come to a group which can be divided only into individuals, and there Division ends. The individual is so called because it cannot be divided by process of logical Division into units. The group next above the individuals was called by the ancient writers on logic "*The lowest species,*" and the largest group of all was called "*The highest genus.*" "Lowest" and "highest" have, of course, no reference to rank or dignity; they mean least and greatest in extension. The "lowest species" in the case of individual human being was said to be the class "Man" and the "highest genus" was said to be substance. Nowadays, however, both logic and anthropology ignore these distinctions and speak of the races of men, and even of the national groups and subdivisions of national groups as if there were no absolutely fixed "lowest species." Still, all are agreed that, when we come to the individual, logical Division has reached its limit. How to proceed from the individual upward to the various higher groups, how to gather particular objects under class names and arrange those classes will be discussed under the head, "Classification" in the Logic of Induction.

Division of a Topic. Allusion was made earlier in this chapter to the use of Logical Division in the arrangement of a topic for study or composition. If the topic can be understood in Extension, and it is convenient to treat it in that way, then, obviously the rules given above apply to it. For example, if the topic were, "The books in my library," the arrangement of those books and the description of them in an essay might very well be prefaced by a division of the volumes into works on theology, works on philosophy, etc., and the requirements of order would be maintained if the above four rules were observed. Sometimes, however, the subject does not readily lend itself to treatment in Extension. An essay on "Cheerfulness," on "Julius Caesar," on "Skating," does not admit of being treated in Extension, though in the case of "Cheerfulness" the various kinds of that quality may be described. In all cases there should be a grouping of our ideas on the topic, these ideas should be grouped in an orderly manner, and the groups should be arranged in an orderly manner. Thus, Rule II applies, because our arrangement should, if the study or composition is to be complete, include all the important subdivisions of the subject. Again, Rule III applies in so far as our ideas, when grouped under several heads, should be co-ordinated and subordinated. For instance, if we arrange what we know about Julius Caesar under the heads "Life" and "Characteristics" and under "Characteristics" arrange "mental traits," "moral traits," "physical appearance," it would be illogical as well as ridiculous to write "Julius Caesar was a man of a high order of intellect, and was only five feet tall." Rule I also applies to the division of topics whenever the subject of our study or composition is capable of being strictly and formally divided. Finally, Rule IV applies to writing, with the proviso that the object or aim of the writer is the paramount principle which determines what is an important basis of division.

Requirement of Rhetoric. In all cases, of course, the requirements of rhetoric demand that strict *logical Division be not obtruded on* *the reader or listener.* There is nothing more tiresome than the detailed announcement of the many points from which the writer or speaker is "about" to treat his subject. But when the art of Logical Division is concealed by means of the rhetorician's art, the effect is certain to be all the more pleasing in proportion as the Division is logically correct. Definiteness and order in the succession of our ideas are a saving of mental effort on the part of our readers, or our audience, and do not interfere with the freedom of treatment which is conceded to be the privilege of the poet, the orator, and the litterateur. Besides, a Division, written out in proper form, as in a synopsis or schema, enables both the speaker or the writer himself and also the reader to survey in a single glance the entire subject and the most salient features of it in their relation to one another.

CHAPTER VI

Judgments and Propositions

A Proposition is the Expression of a Judgment. When in Chapter II we analysed a sample of the process of Reasoning we found that

All colors are physical qualities;
Blue is a color;
Therefore, blue is a physical quality,

is naturally divided first into propositions and secondly into terms. Terms, we saw, are the expression of mental images; propositions, which we are now about to study, are the expression of judgments.

Meaning of "Judgment." "Judgment" is one of those words which, transferred from some external object or institution to an internal fact or condition of the mind, retain part of their original meaning. A judgment was, originally, the action of the judge or magistrate who pronounced the right or the law in a contested question—"*Jus*" "*dicere,*" to declare the law. In the transferred sense it signifies the action of the mind which, after a certain amount of deliberation, pronounces the agreement or difference of two simple elements of thought.

As preliminary to this pronouncement there is required, first, the presence of two mental images, such as those expressed by the words "poet" and "sensitive"; secondly, some kind of comparison or weighing (deliberation) of these two, or of the objects represented by them. But in neither of these stages is there a judgment. Then only is there a judgment when there is a pronouncement of the agreement or difference, that is, when we say, mentally at least, "Some poets are sensitive," "All poets are sensitive," "No poet is sensitive," or "Some poets are not sensitive." *A judgment, then, is the act of the mind by which we pronounce the agreement or difference between two simple elements of thought.*

RASH AND PRUDENT JUDGMENTS. According as the deliberation which precedes the pronouncement is adequate or inadequate the judgment will be "Rash" or "Prudent." Of course, the adequacy of the deliberation depends on the nature and importance of the subject-matter of the judgment. One should deliberate most carefully before judging the moral character of his neighbor, much more carefully than when judging, for instance, his claim to be considered good looking. "Rash" and "prudent" is, however, a distinction that belongs rather to ethics than to logic.

A PRIORI, OR ANALYTIC, AND A POSTERIORI, OR SYNTHETIC, JUDGMENTS. A judgment is "a priori" or "analytic" when the pronouncement is made without appeal to experience, that is, on the evidence that is forthcoming from an analysis of the terms of the judgment. Thus, once I understand what a "whole" is, and what "greater" is, and what a "part" is, I am able to judge, without appeal to experience, that "The whole is greater than its part." An "a posteriori" or "synthetic" judgment is one in which the pronouncement of the agreement or difference is based on experience; for instance, "This rose is red," "Some apples are green." "A priori" is understood to mean "antecedent to experience," while "a posteriori" means "subsequent to, or dependent on, experience." "Analytic" means based on analysis; "synthetic" means resulting from a synthesis, or putting together, of the facts of our experience. Logic recognizes that all "a priori," analytic judgments are, if true, universally and necessarily true, while "a posteriori," synthetic judgments may be universally true or true only in some cases, but never necessarily true. We say "it is in the nature of things" that the whole must be greater than its part; but it is not in the nature of things that all crows are black.

A Proposition Defined. A "Proposition" may be defined as the expression of a judgment. Not every sentence is a proposition. Some sentences express a state of mind without implying a judgment or pronouncement. For instance, a sentence may express a wish or a prayer or a command or an entreaty, as, "Please, hand me that book," "I wish you would close that door," "I beg of you not to make so much noise," "Forward, march." It is only when a sentence is indicative or disjunctive or conditional that it is a proposition; for example, "The book is on the table," "The door is either open or closed," "If you make a noise you will disturb your neighbor."

Categorical, Disjunctive, and Conditional. Thus, we have three kinds of Propositions, categorical, disjunctive, and conditional. A *categorical* proposition simply asserts or denies, as "God is merciful," "Cruelty is not deserving of praise." A *disjunctive* proposition offers two or more alternative assertions or denials, as "He is either honest or dishonest," "He is either not a Catholic or a bad Catholic." A *conditional* proposition neither asserts nor denies simply, but declares the dependence of one assertion or denial on another, as "If he misses Mass on Sunday without a good reason he is not faithful to his duty as a Catholic," or "If Caesar was ambitious he deserved to die." In the last example I do not assert that Caesar was ambitious, nor that he deserved to die, but that if he was ambitious he deserved to die.

Subject, Predicate, and Copula. Taking up now the categorical proposition, we find that it consists of three parts. When we judge, we affirm or deny something of something else. The part of a Proposition containing that which we affirm or deny is called the *predicate,* the part containing that about which the assertion or denial is made is called the *subject,* and the word (or words) which connects them is called the *copula.* Thus:

SUBJECT:	COPULA:	PREDICATE:
God	is	good.
Some men	are not	honest.
Dante	is	a great poet.

This is the form of analysis of Propositions which is adopted in logic. It will be remarked:

(1) *Logical analysis differs from grammatical analysis.* In grammar we distinguish subject and predicate merely; in logic we bring out into explicit expression the copula also.

(2) Sometimes *a sentence must be amplified in order to bring out the copula.* For instance, "The dog barks" should be analysed, "The dog (*subject*) is (*copula*) an animal that barks" (*predicate*).

(3) *The copula "is" does not imply real existence,* as is clear from such examples as "Hercules *is* a mythical person," "The Centaur *is* an animal that never existed." Here the predicate clearly indicates that the copula *"is"* does not imply real existence.

(4) For the purposes of logic *the tense of the verb in the copula should be changed to the present,* that is, to the time when the judgment is made. Thus, "Caesar was ambitious" should be analysed "Caesar is a person who was ambitious."

(5) It is often necessary *to invert the order of words* in a proposition in order to give the subject the first place in the analysis. "Great is Diana of the Ephesians," "Uneasy lies the head that wears the crown," are examples. In a few cases, namely, when the proposition expresses perfect identity, it is a matter of indifference which term is taken for subject and which for predicate. For instance, "Mercury is quicksilver," "Tully is Cicero," "The White House is the Executive Mansion."

Various Kinds of Propositions. Propositions, like judgments, are either *a priori* (*analytic*) or *a posteriori* (*synthetic*). They are further divided according to Quality into *Affirmative* and *Negative.* An Affirmative proposition is one which expresses the pronouncement of the agreement between subject and predicate, as, "Gold is a metal." A Negative proposition is one which expresses the pronouncement of the disagreement, or difference, between the subject and the predicate, as, "Gold is not iron." Usually the copula indicates the quality of a proposition; "is" and "are" are signs of an affirmative, and "is not," "are not," are signs of a negative proposition. Sometimes, however, the negative quality is indicated by "No" placed before the subject, as "No solid is a liquid." According to Quantity, propositions are divided into Universal, Particular, Singular, and Indefinite. A *Universal* proposition is one in which the subject is taken in all its extension, as "All trees are plants," "No mathematician is a poet." A *Particular* proposition is one in which the subject is taken for only part of its extension, as "Some mathematicians are poets," "Some plants are not trees." A *Singular* proposition is one which has for its subject a singular term, as "Judas betrayed his Master," "The Apostle of the Gentiles was born at Tarsus." An *Indefinite* proposition is one in which the quantity is not indicated, as "Books are useful," "Flowers are plants." For the purposes of logic, however, Singular propositions are regarded as universal, because their subject is taken in all its extension even though the extension be a unit. In logic, too, Indefinite propositions do not remain as a separate class, because, if we attend to the meaning, we can determine whether an indefinite proposition should be regarded as universal or particular. Thus, "Books are useful" becomes "Some books are useful," a particular; and "Flowers are plants" becomes "All flowers are plants," a universal.

THE ULTIMATE MEANING OF A PROPOSITION. Philosophers as well as logicians are interested in the question, What is the ultimate meaning of a proposition? In other words, What is *the import* of propositions, What are we talking about when we make an assertion or a denial, Does our Judgment refer to *words*, to *ideas*, or to *things?* Some philosophers answer that the last analysis of a proposition is a matter of *words*. These are the *Nominalists*. They maintain that when we pronounce a judgment our meaning is that the predicate is the name of that of which the subject is the name. Thus, when I affirm that "All trees are plants," I mean that "plants" is the name of the same objects as "trees." Now, no one denies that this is a true statement. But it is not an adequate statement, except in the case of a comparatively small number of propositions, namely, verbal propositions, such as "The White House is the Executive Mansion," "Joan of Arc is the Maid of Orleans," "Saul is Paul." In these cases all that we mean is an identity of names. In other cases, by far the greater number of propositions, we mean much more than this. We mean to go beyond the words to the things which the words signify, and we mean to affirm an identity or disagreement between something besides the words.

Others, disputing the adequacy of the Nominalist interpretation, say that our judgment is concerned not only with words but also with the ideas which the words express, with the mental images, or in many cases, with the *Concepts* for which the words stand. These are the *Conceptualists.* They hold that the ultimate meaning of a Proposition is the assertion or denial of the identity of our mental images. They say, for instance, that the final reference in the proposition, "All trees are plants," is to the agreement of my idea of a plant with my idea of a tree. Here, again, the analysis is true so far as it goes. But, it does not go far enough. If I assert the agreement of two ideas it is because the things which the ideas represent agree also. We should never forget that an idea is only a representation. The thing represented, not the representation, is ultimate. In our study of Religion we learn that when Catholics honor the image or representation of a saint, the image is not the recipient of the honor, but the person whom it represents. This arises from the very nature of a representation. For the function of a representation is to carry our thoughts beyond itself to the thing it represents. When, therefore, we analyse a proposition, and say that it affirms or denies the agreement of two representations—the idea expressed by the subject and the idea expressed by the predicate—we must not stop there, but go beyond the representation to the thing represented.

This is what the *Realists* do. They say that in every proposition (with the exception of the merely verbal propositions noted above) we mean to assert the agreement or difference between two *things* which are represented by the subject and predicate. And the Realists certainly have philosophy and good sense on their side. But, now, we may recall that every term may be read either in Comprehension or in Extension; it may be understood to refer either to the attributes and qualities which it connotes or to the individuals and groups of individuals which it denotes. There remains, therefore, the further question: *Do we refer ultimately to the Comprehension or to the Extension of the subject and predicate when we pronounce a proposition?* Absolutely speaking, there are four possible ways of interpreting a proposition, as indicated in this schema:

SUBJECT:	PREDICATE:	METHOD:
Extension	Comprehension	*Possession*
Extension	Extension	*Inclusion*
Comprehension	Comprehension	*Concomitance*
Comprehension	Extension	*Indication*

That is, (1) *The subject may be understood in Extension and the predicate in Comprehension.* Thus, "All trees are plants" would mean, "All the objects denoted by the term tree possess the attributes comprehended under the term plant." This is called the method of *Possession.* (2) *Both subject and predicate may be read in Extension.* Thus, "All the objects denoted by the term tree are included in the class denoted by the term plant." This is called the method of *Inclusion.* (3) *Both subject and predicate may be read in Comprehension.* Thus, "The attributes comprehended by the term plant accompany the attributes comprehended by the term tree." This is called the method of *Concomitance.* (4) *The subject may be read in Comprehension and the predicate in Extension.* Thus, "The attributes comprehended by the term tree indicate that the object possessing them belongs to the class plant." This is called the method of *Indication.*

If we compare these four methods of reading a Proposition we shall find (1) the first, the method of *Possession* is the most natural. Generally speaking, when we pronounce a judgment we think of the subject in Extension and of the predicate in Comprehension, we think of a class of things possessing certain attributes. (2) The second, the method of *Inclusion,* is the most convenient in the study of propositions and arguments in logic; because, as we shall see in the next chapter, it is convenient to consider Propositions as having both subject and predicate extended. (3) The third, the method of *Concomitance,* is, in most cases, the ultimate philosophical meaning, because the extension reading is based on it. The reason why we place a thing in a certain class is because it has certain attributes. (4) The fourth, the method of *Indication,* is not a natural or spontaneous method of reading a proposition, except in a few rare cases where the attributes of the subject are present in our consciousness and we at the same time think of the predicate as a class. "All that glitters is not gold" may be cited as an instance.

THE REAL EXISTENCE OF THE SUBJECT. Connected with this question of the ultimate meaning of Propositions is the question, "Does a Proposition imply the real existence of its subject?" Before giving a definite answer we must distinguish various spheres of existence. The sphere of history, for instance, is distinct from that of mythology, poetry or fiction. Similarly, the sphere of real existence is distinct from that of imaginary existence. We saw, in dealing with the nature of the copula, that the verb "is" does not necessarily imply real existence. It does, however, imply existence of some kind, either real or imaginary, either in history, mythology, poetry or fiction. It is evident, therefore, that every proposition implies the existence of its subject in some sphere of existence. And, if the proposition is affirmative the implication of the existence of the subject carries with it the implication that the predicate too exists in the same sphere of existence. But in a negative proposition it is possible that the subject exist in the sphere of real existence or of history, while the predicate exists only in the imaginary world of mythology, poetry or fiction. Thus, "Caesar was not Mars." Caesar is a real historical person, whereas Mars is a mythological person.

THE STUDY OF PROPOSITIONS IN LOGIC. Propositions are, therefore, to the act of judgment what terms are to mental images. They express our judgments. Our study of this part of logic will, naturally, include an examination of the logical determinants of the thought-value of propositions for the purpose of showing how consistency and coherence are to be secured in our judgments.

CHAPTER VII

The Four Types of Propositions

A, E, I, AND O. In the preceding chapter we saw that Propositions are divided according to *Quality* into *Affirmative* and *Negative,* and according to *Quantity* into *Universal, Particular, Singular,* and *Indefinite.* We saw, moreover, that the last two classes need not be retained, because they can be reduced to Universal and Particular. Combining now the distinction according to Quantity with the distinction according to Quality, we have four kinds of Propositions: Universal Affirmative, Universal Negative, Particular Affirmative, and Particular Negative. It is usual to designate these by the first four vowels of the alphabet, so that *A stands for Universal Affirmative, E for the Universal Negative, I for the Particular Affirmative, and O for the Particular Negative.* "All crystals are solids" is a Universal Affirmative, an A proposition. "No crystals are liquids" is a Universal Negative, an E proposition. "Some solids are crystals" is a Particular Affirmative, an I proposition. And "Some solids are not crystals" is a Particular Negative, an O proposition.

When is a Term Distributed? Going back, now, to the method of reading a proposition by "inclusion," that is to say, by taking both subject and predicate in extension, let us inquire into the extension of the subject and the predicate in the case of each of these four types of proposition. When a term is taken in all its extension it is said to be *distributed,* when it is taken in part of its extension it is said to be *undistributed.*

A Universal Proposition Always Distributes its Subject. It is obvious, in the first place, that in a Universal Proposition the Subject is distributed. That is to say, whatever is affirmed or denied of the Subject is affirmed or denied of the Subject in all its extension. When I affirm that "Every virtuous action is praiseworthy," I speak of the whole class of virtuous actions; when I assert that "No mathematician is a literary person," I mean to exclude the entire group denoted by the Subject from the group of persons denoted by the Predicate.

A Particular Proposition Never Distributes its Subject. In the next place, it is equally obvious that in a Particular Proposition the Subject is not distributed. Whatever is affirmed or denied of "some" of the Subject is affirmed on denied of a part of the extension of the Subject. "Some habits are praiseworthy," "Some mathematicians are literary persons," are instances of particular propositions in which it is evident that the Subject is taken for part, and only for part, of its extension.

An Affirmative Proposition Never Distributes its Predicate. We come, next, to the Predicate. In an affirmative proposition, whether it is Universal or Particular, we affirm the inclusion of the class denoted by the Subject in the class denoted by the Predicate. In order that this inclusion be true it is not necessary that the extension of the Subject coincide with all the extension of the Predicate, but only with part of it. For instance, when I say "All trees are plants," I affirm that all the extension of "trees" coincides with some of the extension of "plants," and when I affirm that "Some trees are beautiful" I mean to identify part of the extension of "trees" with part of the extension of "beautiful (things)." It is true, there are cases in which all the extension of the predicate coincides with all the extension of the subject. This is the case in definitions and in identical propositions, as "All triangles are three-sided figures," "St. Paul is the Apostle of the Gentiles." But, this is an exceptional relation of Subject and Predicate, and not implied in every affirmative proposition.

A Negative Proposition Always Distributes its Predicate. Finally, still speaking of the Predicate, in Negative Propositions the Predicate is always distributed. When we exclude the Subject, whether in part or totally, from the Predicate, we exclude it from all the Predicate. Thus "No tree is a mineral" excludes the totality of the Subject from the whole class denoted by the Predicate. "Some mathematicians are not philosophers" excludes part of the Subject from all the Predicate.

Rules for the Distribution of Subject and Predicate. We are now in a position to formulate the following set of rules:

I. *A Universal proposition always distributes its Subject.*

II. *A Particular proposition never distributes its Subject.*

III. *An Affirmative proposition never distributes its Predicate.*

IV. *A Negative proposition always distributes its Predicate.*

These are not Rules arbitrarily laid down by the logician. They simply sum up the foregoing analysis of the various types of propositions. As convenient statements of the results of that analysis they should be borne in mind in our subsequent study of the laws of logical consistency and logical inference.

Application of These Rules. Applying these Rules to the four types of Propositions, we see at once that A, the Universal Affirmative, distributes its Subject only; E, the Universal Negative, distributes both Subject and Predicate; I, the Particular Affirmative, distributes neither Subject nor Predicate; O, the Particular Negative, does not distribute its Subject, but distributes its Predicate. For the purposes of ready reference, we may, therefore, draw up this schema:

	SUBJECT:	PREDICATE:
A	*Distributed*	*Not-distributed*
E	*Distributed*	*Distributed*
I	*Not-distributed*	*Not-distributed*
O	*Not-distributed*	*Distributed*

These four types of Propositions may be said to be the forms into which our judgments are ordinarily cast. It is possible, however, to cast our judgments into extraordinary forms, and, by ingenious distinctions, to add to these four types several others more or less artificial.

QUANTIFICATION OF THE PREDICATE. The most celebrated attempt in this line is that of the Scottish logician, Sir William Hamilton (1788-1856)[1] whose *Doctrine of the Quantification of the Predicate* finds mention in all our text-books on logic. Hamilton commences by postulating

[1]Hamilton was well read in the works of medieval logicians, and probably knew that the famous Spanish Cistercian, John Caramuel (1606-1682), had worked out a similar scheme of Quantification of the Predicate in his *Logica Vocalis*, published among his *Opera*, Frankfurt, 1654. For life of Caramuel see *Catholic Encyclopedia*, Art. "Caramuel y Lobkowitz."

that "We may be allowed to state explicitly in language all that is contained implicitly in thought."[2] He then asserts that in thought we quantify not only the Subject but also the Predicate; we sometimes judge that "All trees are (some) plants," but we sometimes judge that "All triangles are all three-sided figures." Carrying out this principle, he finds that there are eight types of propositions, namely:

From A	All S is all P All S is some P
From E	No S is any P No S is some P
From I	Some S is some P Some S is all P
From O	Some S is not any P Some S is not some P

Among the advantages, which, according to Hamilton, follow from the quantification of the predicate is this: All logical propositions are reduced to definite quantitative relations, so that we can write the mathematical sign of equation between Subject and Predicate in every case. On the other hand, the objections to this arrangement are not to be overlooked. Hamilton's Postulate is, to say the least, ambiguous. What he means, apparently, is not that "we may be allowed to state explicitly in

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[2] *Lectures on Logic*, I, 114.

language all that is contained implicitly in thought," for there is much that is implied in every judgment that cannot be explained even in thought except through processes of inference. He must mean that "we may be allowed to state explicitly in language all that is contained explicitly in thought." But do we, in thought, explicitly quantify the predicate? This is the test of Hamilton's theory. If we examine our own judgments we shall find that:

(1) Whenever we think accurately and clearly about any subject of a judgment, we quantify that subject, we think of it as "all" or "every" or "some" or "no" S.

(2) We seldom quantify the predicate. Because we think of it, in most cases, in comprehension, and refer to the class, or extension, aspect of it only for the purposes of formal logic.

If, for example, the Subject of the judgment is "metals" and the predicate "solid," I go over in my mind the various kinds of metal, gold, silver, iron, mercury, etc., and at the same time I think of the quality which we call solidity. That is to say, I think of the extension of the subject, and, since mercury is not a solid, I make the affirmation of solidity of "some metals." But I do not think explicitly about the class "solids" at all. I do not refer to the question whether metals are all solids or only some solids. It matters little whether there are or are not other solids besides metals. All I think about, explicitly, is the attribution of the quality "solidity" to some of the class of objects denoted by the term "metals."

Usual Signs of Quantity. The signs by which the quantity of a proposition is usually indicated are: "This" to designate a singular proposition, as "This rose is red"; "Each," "Every," "All," to designate a Universal, as "Each day has its duty," "Every deed has its reward," "All tyrants are cowards"; "No" to designate a Universal Negative, as "No patriot loves self more than country"; "Some" to designate a particular proposition, as "Some lions are tame," "Some poets are not musicians."

Other Marks of Quantity. There are, however, other marks of quantity which need to be explained:

(1) *Numerically definite statements,* as "One third of the population of this city is Catholic." This is, for the purposes of logic, a particular proposition, and so far as the rules of logic are concerned, is equivalent to the proposition, "Some of the people of this city are Catholics." However, it implies also an examination into the religious beliefs of all the inhabitants of the city, and, therefore, entitles us to assert that "Two-thirds of the population of this city are

non-Catholics," or in formal logic, "Some of the people of this city are non-Catholics."

(2) "Any" is *an indication of Universality.* "Any schoolboy could tell you" means "Every schoolboy could tell you."

(3) "A few" or "few" is a sign of *particular quantity.* For the purposes of logic, it is equivalent to "some." Strictly speaking, however, it has a more definite meaning than "some." "Some" means "less than all," while "few" means "a small number," "perhaps less than half." If I assert that "Some Democrats are expansionists" I may mean that one or two Democrats share the views of the expansionists, or I may mean that all Democrats except one or two share those views. But, if I say "Few Democrats are expansionists," I must mean "a small number," at least less than half.

(4) "Most," like "few," is equivalent to "some." However, like "few," it may be interpreted strictly; it means a larger number, "at least more than half." It would, for instance, be true to say that "Some metals are liquid," but it would not be true to say "Most metals are liquid," because in normal conditions of temperature, mercury is the only metal, so far as we know, that occurs in a liquid state.

(5) "Hardly any" is equivalent to "few" or to "most" followed by a negative. "Hardly any Catholic historian agrees with Gibbon"

means "Few Catholic historians agree with Gibbon," or "Most Catholic historians do not agree with Gibbon."

(6) *"Except," "alone," etc., limit the extension of the subject,* and place the proposition in the class known as *exponible* propositions. The meaning of such propositions is brought out by *expounding* them. Thus, "All the Apostles, except Judas, remained faithful" means (a) "Peter, James, etc., remained faithful"; (b) "Judas did not remain faithful." "Protestants alone were eligible to parliament" means (a) "All Protestants were eligible to parliament"; (b) "No one who was not a Protestant was eligible to parliament."

To the class *exponible* belong all propositions which must be resolved into two or more propositions in order to bring out their meaning. Thus *inceptive* and *desitive* propositions are exponibles which refer to the beginning or the end of some action or condition. "At six o'clock he began to study" means that before that hour he was not studying and that after six o'clock he was studying. Similarly, "At half past one he stopped studying" is resolvable into two statements.

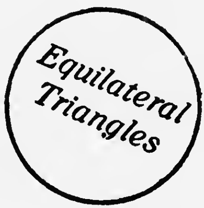
THE INCLUSIVE MODE OF READING PROPOSITIONS. As has been said more than once in the course of this lesson, logic, ignoring the more definite numerical designations, reduces all quantity to two kinds, Universal and Particular, and in the theory of Opposition and Inference and in the laws of syllogistic reasoning treats every proposition as belonging to one of the four fundamental types, A, E, I and O. It should be borne in mind that the mode of reading propositions here adopted, namely, the mode of inclusion, does not supersede the mode of concomitance, by which propositions are read in comprehension. The "extensive" or "inclusive" mode is only one mode of interpretation. Yet it is the one which gives us the best results in formal logic, in treating of Opposition, Conversion and Inference, although in some cases it may seem stilted and unnatural. For instance, the logical form of the sentence, "The train leaves at one o'clock" would be "The train is something that leaves at one o'clock." Clumsy as this form is, it is the only form which, as we shall see in the ensuing chapters, lends itself to treatment in logic.

CHAPTER VIII

Propositions Represented by Diagrams

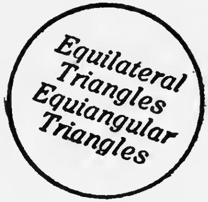
Logicians decide that, although, as we saw in Chapter VI, there are four possible ways of reading a proposition, according as we read (1) *Both subject and predicate in extension,* (2) *Both subject and predicate in comprehension,* (3) *Subject in extension and predicate in comprehension,* or, (4) *Subject in comprehension and predicate in extension,* the first of these modes of reading a proposition is the most convenient for the logician's purpose. Consequently we shall adopt that mode without, however, implying that the others are untrue or that they have not their use in psychology or in philosophy. We shall understand the subject to be a group or class of objects or persons, and the predicate, similarly, to be a group or class. Whatever agreement or difference is expressed by the proposition will be understood as an agreement or difference between the extension of the subject and the extension of the predicate.

In case of the A proposition, the Universal Affirmative, it may happen that the extension of the subject exactly coincides with that of the predicate. For instance, when we say "All equilateral triangles are equiangular triangles," the extension of the subject is neither wider nor narrower than the extension of the predicate. If, therefore, the extension of the subject is represented by means of a circle, thus,



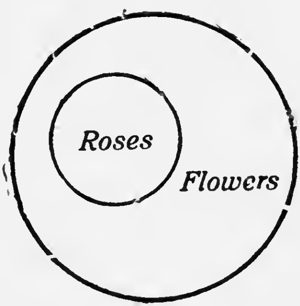
and the extension of the predicate is represented by another circle, thus,

it is evident that the two circles are identical, that we may, in other words. place both the extension of the subject and that of the predicate in the same circle, as in the following diagram:



All the extension of the subject and all of the extension of the predicate are within the circle; none of the extension of either is outside the circle.

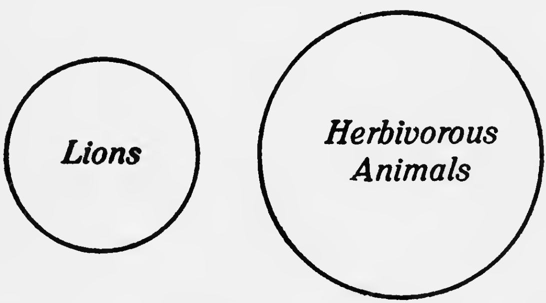
This is, however, an exceptional case of the A proposition. Usually the extension of the subject is less than that of the predicate, and the true state of their relation is indicated by two circles, the smaller, representing the subject, being completely included in the larger circle, which represents the predicate. For example, the following diagram:



represents the proposition "All roses are flowers."

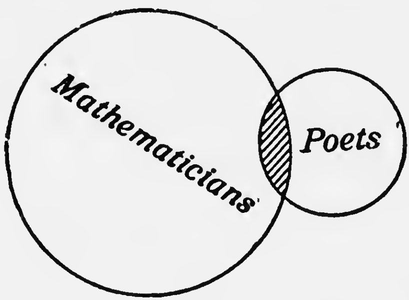
All the extension of the subject coincides with part of the extension of the predicate. Outside the circle marked "Roses" are violets, pansies, and all other groups included under the extension of the term "Flowers." This is the type of an A proposition, and the relation between subject and predicate which it affirms is that of *Total Inclusion* of the former in the latter.

In the E proposition we have a case of *Total Exclusion.* The Universal Negative proposition places all the extension of the subject outside the extension of the predicate. When we judge that "No lion is a herbivorous animal" we exclude all the extension of the term "lion" from the extension of the term, "herbivorous animal." The diagram, therefore, which represents an E proposition will consist of two circles,

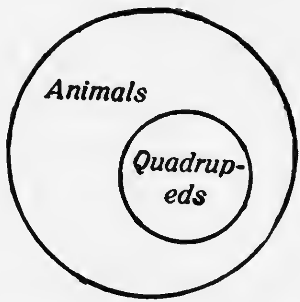


which do not come into contact at all, but are completely excluded from each other. If the circles partly coincided, there would be some lions which are herbivorous, and it would no longer be true that "No lions are herbivorous animals."

In the Particular Affirmative, the I proposition, part of the extension of the subject is included in the extension of the predicate. This condition is, therefore, one of *Partial Inclusion.* Thus, the proposition, "Some mathematicians are poets," may be represented by the diagram,

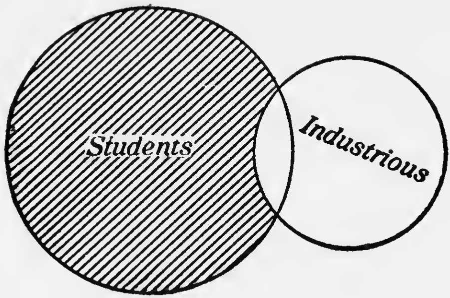


In the shaded portion of the diagram is represented that part of the extension of the subject which is included in the extension of the predicate. There, for the purpose of representing the truth visually, we place those mathematicians who are at the same time poets. In the clear portion of the circle representing the subject are those mathematicians who are not poets, and in the clear portion of the circle representing the predicate are those poets who are not mathematicians. Here, too, we may have to deal with an exceptional case, as when we judge that "Some animals are quadrupeds." In this case, the diagram representing all the truth of the proposition would be



the fact being that there is a coincidence between part of the extension of the subject and all the extension of the predicate. This, however, is not the usual form of the I proposition; in fact, it is the diagram of the A proposition, and the more natural expression of the state of affairs here represented would be "All quadrupeds are animals."

In the Particular Negative, or O proposition, the extension of the subject is, in part, excluded from the extension of the predicate. It is a case of *Partial Exclusion.* Thus, the proposition, "Some students are not industrious," may be represented graphically by the following diagram:



The shaded portion of the circle to the left represents the students who are not industrious; the remainder of that circle represents industrious students, and the right hand portion of the other circle represents industrious persons who are not students.

Going back now to the four rules laid down on page 96 in regard to the distribution of the subject and predicate of a proposition, we realize very readily by the aid of these diagrams that:

I. *A Universal proposition always distributes its subject.* For, whether it is affirmative or negative, it takes the subject in all its extension. If it is affirmative, it *includes all* the subject in the extension of the predicate, and if it is a negative, it *excludes all* the subject from the extension of the predicate.

II. *A Particular Proposition never distributes its subject.* If it is affirmative, it *includes* *part* of the subject in the extension of the predicate, and if it is negative, it *excludes part* of the extension of the subject from the extension of the predicate.

III. *An Affirmative Proposition never distributes its predicate.* For, apart from the exceptional case of the A proposition in which the two circles coincide, and the equally exceptional case in which the I proposition has for its predicate a whole class, represented by a complete circle, the identity or coincidence is always between the subject and *part* of the extension of the predicate. The shaded portion of the predicate is the extension of the predicate, *as used in that proposition.*

IV. *A Negative Proposition always distributes its predicate.* When we deny, we exclude the subject, either all of it (in the E proposition) or part of it (in the O proposition) from *all* the extension of the predicate. The exclusion may be partial or total, as far as the subject excluded is concerned; it is always total, as far as the predicate is concerned, from which the subject is excluded.

These diagrams are generally referred to as the *Eulerian Diagrams,* because they were introduced into the teaching of logic by the celebrated Swiss logician and mathematician, Euler (1707-1783).

CHAPTER IX

The Opposition of Propositions

CONSISTENCY AND INCONSISTENCY. One of the purposes of logic, as was pointed out in the first chapter,[1] is "so to direct the mind * * * as to enable it to attain * * * *consistency*" in the process of judging, which is one of the processes subsidiary to reasoning. We all know, in a general way, what consistency and inconsistency are. We can tell, in most cases, whether two statements or judgments are consistent or inconsistent with each other. We are aware, for instance, that a speaker who says that "All the members of the legislature are lawyers," and, a moment later, that "Some members of the legislature are not lawyers," contradicts himself. The two assertions are obviously inconsistent. It is the aim of logic to show, by means of the doctrine of Opposition and Conversion, just how propositions are opposed to one another or agree with one another or may be inferred from one another.

LOGICAL OPPOSITION. Two propositions which deal with different subject matters cannot be

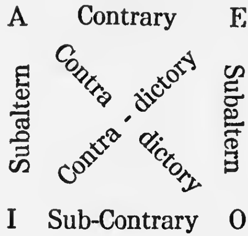
[1] Page 17.

said, strictly, to be opposed to each other, or to be consistent with each other. When two judgments fall in different regions of thought, there is no logical relation between them. Thus, there is no logical relation between "All lions are carnivorous" and "Some plants are perennial." Neither is there any logical relation between "All lions are carnivorous" and "All lions are felines," although, as we shall see later, a logical relation may arise out of the combination of these two in a process of inference. In order to have a relation of opposition between two propositions, the propositions *must have the same Subject and the same Predicate.*

But two propositions which have the same Subject and the same Predicate may differ in Quantity only, one being universal and the other particular, or in Quality only, one being affirmative and the other negative; or, they may differ both in Quantity and in Quality.

OPPOSITION DEFINED. We are now in a position to define what we mean by Opposition of propositions: *Two propositions are said to be opposed when, having the same subject and the same predicate, they differ in quantity or quality or both.*

SQUARE OF OPPOSITION. If, bearing this definition in mind, we arrange the four fundamental types of propositions, namely, A, E, I, O, in a square:



we shall find that the greatest opposition exists between A and O, and between E and I, for in both cases there is a difference both of quantity and of quality.

Contradictory Propositions. This species of Opposition is called *Contradiction. Two propositions are, therefore, contradictory, when, having the same subject and the same predicate, they differ both in quantity and in quality.* The contradictory of a Universal Affirmative (A) is a Particular Negative (O), and the contradictory of a Universal Negative (E) is a Particular Affirmative (I). Thus, the contradictory of "All the Athenians are lovers of wisdom" is "Some of the Athenians are not lovers of wisdom," and the contradictory of "None of the Romans excelled in philosophy" is "Some of the Romans excelled in philosophy."

Contrary Propositions. If we compare A and E, we shall find that, while they have the same quantity, both being universal, they differ in quality, A being affirmative and E being negative. This relation is called *Contrariety. Two propositions, therefore, are said to be contrary when, having the same subject and the same predicate, and both being universal, they differ in quality.* Thus the contrary of "All the Chinese are industrious" is "No Chinaman is industrious."

Sub-contrary Propositions. Next, between I and O, there exists a relation similar to that which exists between A and E. I and O, both being particular, differ in quality, I being affirmative and O, negative. This species of Opposition is called *Sub-Contrariety. Two propositions are said to be sub-contrary when, having the same subject and the same predicate, and both being particular, they differ in quality.* Thus, the sub-contrary of "Some poets are philosophers" is "Some poets are not philosophers."

Subaltern Propositions. Finally, when we compare A and I on the one hand and E and O on the other, we find that in each pair there is an agreement as to quality, but a difference of quantity. This species of Opposition is called *Subalternation. Subalternates*[1] *are, therefore, two propositions which, having the same subject and the same predicate, and agreeing in quality, differ in quantity.* Thus "Some books are useful" is the subaltern of "All books are useful."

[1]Strictly speaking, the Particulars, I and O, are *Subalterns*, the Universals, A and E, are *Subalternants.*

Four Kinds of Opposition. Summing up, now, the results of this examination of the four kinds of opposition, we find that:

(1) *Contradictories,* A and O, E and I, differ both in quantity and in quality.

(2) *Contraries,* A and E, differ in quality.

(3) *Sub-contraries,* I and O, differ in quality.

(4) *Subalterns,* A and I, E and O, differ in quantity.

Incompatibility of Contradictories. The next question is that of compatibility. There is a very fundamental law of thought, self-evident to the average mind, and never called in question by the majority of healthy-minded people. It is generally enunciated in this form: A thing cannot be and not be at the same time. This is called *The Principle of Contradiction.* If we apply it to the propositions which we call contradictory, we shall find at once that such propositions cannot both be true. If "All S is P" is true, then "Some S is not P" cannot be true at the same time. If it is true that "All metals are fusible" it cannot be true that "Some metals are not fusible." Neither can both be false. For the attribute connoted by the predicate "P" either belongs to all "S," or it does not: If it does, the A proposition is true, if it does not, the O proposition is true.

Contradictories cannot both be true.
Contradictories cannot both be false.

Incompatibility of Contraries. Contrary propositions cannot both be true. For if the predicate is affirmed of every member of a class, it surely cannot be denied of every member of that same class. If it is true that "All the apples in this basket are ripe," it cannot be true that "None of the apples in this basket are ripe." However, Contraries may both be false. For the predicate may be applicable to some members of the class and not applicable to others. It may be that some, and some only, of the apples in the basket are ripe. In that case "All the apples in the basket are ripe," and "None of the apples in the basket are ripe" are both false.

Contraries cannot both be true.
Contraries may both be false.

Incompatibility and Compatibility of Subcontraries. If we consider, now, subcontrary propositions we shall find that they cannot both be false. For if I is false, its contradictory, E, is true; and if O is false, its contradictory, A, is true. But we have just seen that A and E, being contraries, cannot both be true. Therefore, I and O cannot both be false. For example, if it is false that "Some learned men are sociable" it cannot be false that "Some learned men are not sociable." Subcontraries, however, may both be true; indeed, as a general rule, if one is true the other is also true. For the predicate which is affirmed of part of the extension of the subject may, generally speaking, be denied of the remainder of the extension of the subject. When we say "Some of the books on this table are bound" it may be true, and generally is true, that "Some of the books on this table are not bound."

Sub-contraries cannot both be false.

Sub-contraries may both be true.

COMPATIBILITY AND INCOMPATIBILITY OF SUBALTERNS. Finally, if we compare subaltern propositions we shall find that the truth of the universal implies the truth of the particular; if it is true that "All poets are inspired by the Muses," it is true that "Some poets are inspired by the Muses," and if it is true that "No poet is inspired by the Muses" it is true that "Some poets are not inspired by the Muses." On the other hand, if the particular is false, the universal is false: for instance, if it is false that "Some of the first Christians were socialists" it is certainly false that "All the first Christians were socialists," and if it is false that "Some Catholics are not true patriots," it is false that "No Catholic is a true patriot." Hence, we have the following rule: *The truth of the particular follows from the truth of the universal and the falsity of the universal follows from the falsity of the particular.*

Exercise. As an exercise in the application of the foregoing rules we may make several successive suppositions in order to see what follows.

(1) If A is true,
  (a) O is false (Contradictory)
  (b) E is false (Contrary)
  (c) I is true (Subaltern)
(2) If A is false,
  (a) O is true (Contradictory)
  (b) E is doubtful (Contrary)
  (c) I is doubtful (Subaltern)
(3) If E is true,
  (a) I is false (Contradictory)
  (b) A is false (Contrary)
  (c) O is true (Subaltern)
(4) If E is false,
  (a) I is true (Contradictory)
  (b) A is doubtful (Contrary)
  (c) O is doubtful (Subaltern)
(5) If I is true,
  (a) E is false (Contradictory)
  (b) A is doubtful (Subaltern)
  (c) O is doubtful (Subcontrary)
(6) If I is false,
  (a) E is true (Contradictory)
  (b) A is false (Subaltern)
  (c) O is true (Subcontrary)
(7) If O is true,
  (a) A is false (Contradictory)

(b) E is doubtful (Subaltern)
(c) I is doubtful (Subcontrary)

(8) If O is false,
(a) A is true (Contradictory)
(b) I is true (Subcontrary)
(c) E is false (Subaltern)

COMPARISON OF RESULTS. If we compare these results we reach some very interesting conclusions, for instance,

(a) We find that 1 and 8, 2 and 7, 3 and 6, and 4 and 5 respectively, give the same results. That is to say, *the truth of the universal gives us the same results as the falsity of the particular of the opposite quality.*

(b) We find that "doubtfuls" occur only when we suppose a universal to be false, or a particular to be true. *We get more definite results when we suppose a universal to be true or a particular to be false.* This leads to the reflection that it *takes more to prove a universal to be true* than it does to prove a particular to be true, and it is easier to prove a universal to be false than it is to prove a particular to be false. For example, if one were discussing the relation between "literary ability" and "business sense" it would be easier to prove that "Some literary men have no business sense," than to prove that "All literary men are lacking in business sense." But it would be more difficult to disprove the particular "Some literary men are lacking in business sense'' than to disprove the universal ''All literary men are lacking in business sense.''

Practical Consequences. It follows from this that when in a debate or discussion we wish to disprove the proposition which an opponent defends, *it is enough to prove the contradictory of that proposition.* It is not necessary, it is not advisable to attempt, because it is not always possible, to prove the contrary of our opponent's contention. If, for example, our opponent defends the proposition ''All works of fiction should be excluded from public libraries,'' in order to refute him it is sufficient for us to prove the contradictory, namely, ''Some works of fiction should not be excluded from public libraries.'' If we go farther, and undertake to prove the contrary, namely, ''No work of fiction should be excluded from public libraries'' we undertake to prove too much, and leave the way open to our adversary to show that there are some works of fiction which should be excluded from public libraries, a task which is by no means difficult.

At the same time, we should be careful not to err in the opposite direction *by proving too little.* In order to refute a proposition it is not sufficient to establish the subaltern or the subcontrary. The assertion ''All works of fiction should be excluded from public libraries'' is not refuted by establishing its subaltern ''Some works of fiction should be excluded from public libraries," and the proposition "Some Catholics are illiterate" is not refuted by establishing the subcontrary "Some Catholics are not illiterate."

Practical Rule. The rule should be, *to refute a proposition it is sufficient to establish its contradictory,* and even when it is possible to establish the contrary, it is not advisable to do so, unless one is perfectly certain that the adversary cannot retort by disproving the contrary.

Opposition of Singular Propositions. Singular or individual propositions were reduced to the class universal. The proposition, for example, "Socrates is wise" is an A proposition. In cases like this, the contrary and the contradictory are the same, namely, "Socrates is not wise." Some logicians, however, introduce here what they call the *material contrary and contradictory,* so that in the example given, the contrary would be "Socrates has not a grain of sense," and the contradictory would be "In some instances, sometimes, or to a certain extent, Socrates is not wise." These are called material contrary and material contradictory because they are not derived from the form of the thought but from the matter.

Difference Between Contrariety and Contradiction. Whenever we have to deal with

Opposition, whether formal or material, we shall find that the difference between the contradictory and the contrary of a given proposition is *the difference between irreconcilable opposition and opposition which goes so far that it is hardly true opposition at all.* Between two contradictories there is no compatibility, whereas in the case of two contraries, there is this much compatibility that they may both be false. Again, *between two contradictories there is no tenable medium,* while in the case of two contraries there is often a medium wherein the truth lies. The propositions "All colors are physical phenomena" and "Some colors are not physical phenomena" are utterly incompatible, and offer no possible alternative; one must be true and the other must be false. If, however, we compare two contraries, such as "All philosophers are poets" and "No philosopher is a poet," we find that both are false, the truth being that "Some philosophers are poets" and "Some philosophers are not poets."

CHAPTER X

Conversion of Propositions

COMMON SENSE AND INFERENCE. In the same way as we all realize that two contradictory propositions are opposed to each other, we realize too that sometimes one proposition implies, or includes, another. It does not require much training in logic to perceive that "All spiritual beings are immortal" somehow warrants us in saying "Therefore, no spiritual being is mortal." In point of fact, people who do not even know that there is a science of logic are constantly inferring or deducing one proposition from another. It is the task of logic to inquire briefly into the nature of inference and, adhering always to the extension method of reading propositions, to draw up a set of rules for inference and point out the various kinds of valid inference.

WHAT IS INFERENCE? Inference in general may be described as *a process by which the truth implicitly contained in a premise or in premises is brought out explicitly, or made known.* When there is only one premise the inference is *immediate.* For example, it is evident that one may argue:

"Some Democrats are Imperialists;
Therefore, Some Imperalists are Democrats."

When there are two or more premises, the inference is *mediate,* as

All colors are physical qualities;
Blue is a color;
Therefore, Blue is a physical quality.

At present, we are concerned only with immediate inference, of which Conversion is the most important species.

CHANGE OF QUALITY AND QUANTITY. In dealing with propositions in logic, we take for granted that any proposition can have its terms transposed and its quantity and quality changed as long as the change of quality is compensated for, and the change of the quantity and transposition of terms do not necessitate any term being distributed in the new proposition which was not distributed in the proposition with which we started. The whole theory of Conversion resembles the theory of algebraic equations so far as the quality is concerned. If $a+b=c$, then $a-(-b)=c$. If "All angels are immortal" then "No angel is mortal." As far as the distribution of terms is concerned, the theory of Conversion may be compared to the practice of banking. In the proposition with which we start, we deposit, so to speak, two terms, on which we can validly draw as long as we do not overdraw the amount deposited. We overdraw when a term that was not distributed in the first proposition is taken distributively in the new proposition. Thus, if one were to infer from "All trees are plants," that "All plants are trees" the inference would be invalid, because "plants" is taken distributively in the second proposition (the conclusion) while it was not used distributively in the first proposition (the premise), for there it is the predicate of an affirmative proposition.

Rules of Conversion. We may formulate the foregoing remarks in two rules which apply to Conversion in general:

I. *Every change of quality must be compensated for by the introduction of a negative in the Predicate.*

II. *No term may be taken distributively in the conclusion which was not distributed in the premise.*

Simple Conversion. The simplest kind of Conversion consists in transposing the subject and predicate without making any change in the quantity or quality. "No Christian is an atheist; therefore, no atheist is a Christian." Similarly "Some solids are minerals; therefore, some minerals are solids." In these instances an E and an I proposition are converted simply, that is to say, from the original premise a new proposition (the conclusion) is inferred, having for subject the predicate, and for predicate the subject of the original proposition, without any change in the quantity or the quality. But, if we try to convert an A or an O proposition in this way we shall find that the process is not valid. From "All horses are quadrupeds" it does not follow that "All quadrupeds are horses" because the term "quadrupeds," distributed in the conclusion (being the subject of a universal proposition) was not distributed in the premises (where it was the predicate of an affirmative proposition). And from "Some Chinese are not heathens" it does not follow that "Some heathens are not Chinese," because "Chinese" is taken distributively in the conclusion, whereas it was not distributed in the premise.

Simple Conversion *is, therefore, a process of immediate inference in which from a given proposition we infer another, of the same quantity and quality, having for subject the predicate, and for predicate the subject of the original proposition.* Only E and I can be converted simply.

Conversion per Accidens. In the case of the A proposition, we cannot, as was shown above, infer an A proposition by simple conversion. But we can infer an I proposition by a process which is called *Conversion per Accidens,* or

Conversion by Limitation. Thus, from the proposition "All cathedrals are churches" we may infer "Some churches are cathedrals." The predicate "churches" in the premise is not distributed; therefore, according to Rule II, it may not be distributed when it is made subject in the conclusion, which means that the conclusion must be particular. An E proposition could also be converted per accidens; from "No quadruped is a bird" we could infer "Some birds are not quadrupeds"; but, since, by simple conversion we may infer "No bird is a quadruped," the result of the conversion per accidens of E is useless.

CONVERSION PER ACCIDENS *is, consequently, a process of immediate inference by which from a universal proposition we infer a particular of the same quality, having for subject the predicate and for predicate the subject of the original proposition.* A and E may be converted per accidens.

OBVERSION. According to what has been said in explanation of Rule I, we may change the quality of any proposition if we make compensation for the change by substituting for the predicate its contradictory. This applies to all kinds of propositions. From "All the representatives are honest" it follows that "None of the representatives is dishonest." From "None of the apples is ripe" it follows that

"All the apples are unripe." From "Some historians are untruthful" it follows that "Some historians are not truthful." From "Some chemical compounds are not stable" it follows that "Some chemical compounds are unstable." This process is called *Obversion.* It leaves the subject and predicate in the same positions and does not change the quantity of the proposition. It merely substitutes for the predicate the contradictory of the predicate and changes the quality of the proposition. Care should be taken in dealing with a certain class of predicates which have a contrary as well as a contradictory. "He is not scrupulous," should not be obverted into "He is unscrupulous," because "unscrupulous" is the contrary, whereas "not-scrupulous" is the contradictory of "scrupulous." Similarly "poor" is the contrary of "rich," "harmful" is the contrary of "beneficial."

Obversion, then, *is a process of immediate inference in which from a given proposition we infer another of the same quantity, but of different quality, having for subject the subject and for predicate the contradictory of the predicate of the original proposition.* A, E, I and O may be obverted.

Contraposition. The process called *Contraposition* is simply a combination, or rather, the successive use, of Obversion and Simple Conversion. After we have obtained the obverse we may, in case it is an E or an I, convert it simply, and the result will be the contraposit of the original proposition. Thus, "All cowards are cruel" becomes by obversion "No coward is not-cruel," from which, by simple conversion, we get "No person who is not cruel is a coward." This is the contraposit of "All cowards are cruel." By a similar process, an O proposition may be contraposited. Thus, "Some apples are not ripe" by obversion becomes "Some apples are unripe," which, by simple conversion, gives "Some unripe (fruit) are apples." This is the contraposit of "Some apples are not ripe."

CONTRAPOSITION is, therefore, *a process of immediate inference in which from a given proposition we infer another, of different quality, having for subject the contradictory of the predicate and for predicate the subject of the original proposition.* Only A and O can be contraposited. Because, if we start to contraposit E or I we get by Obversion A or O respectively and the process must end there, because A and O cannot be converted simply.

SUMMARY OF RESULTS. Besides these four kinds of conversion, namely Simple Conversion, Conversion per Accidens, Obversion and Contraposition, there are other processes which are possible and valid, but of very little practical use. The following table shows the kinds of propositions to which each of these species of conversion is applicable:

Simple Conversion	E and I
Conversion per accidens	A and E
Obversion	A, E, I, O
Contraposition	A and O

Conversion is True Inference. Some logicians doubt whether conversion should be regarded as a species of inference. They maintain that "it brings out no new fact," and that, consequently, it is at most a process of interpretation, not a true inference. It is important, therefore, once for all, to clear up our ideas about inference and its relation to truth and knowledge. Inference, whether mediate or immediate, *adds to our knowledge,* by bringing out explicitly in the conclusion a relation which was only implicitly contained in the premise or premises. *It does not add to truth.* It need not add any new fact. Inference means necessary implication: that is to say, the truth of the premise implies the truth of the conclusion, but we do not know the conclusion to be true until we have explicated, or evolved, it from the premise. Those who follow Mill in maintaining that conversion is not inference, but merely a change of expression, should, if they were consistent, follow him, also, when he rejects the syllogism and maintains that it is not a form of inference. Where there is a transition from one proposition to another, where one proposition is asserted by virtue of the truth of the other, where there are two distinct judgments one implying the other, there is that transition from a known truth to a truth hitherto unknown which we call inference.

Other Kinds of Immediate Inference. In addition to conversion there are a few kinds of immediate inference which call for mention in this portion of logic. If, in algebra, we are warranted in inferring that, because $a=b$, $a+x=b+x$, the same quantity x, being added to both sides of the equation, so also in logic if all S is P, then all $S+x=P+x$, x in this case being some qualification or determinant added to the subject and the predicate of the proposition. If "All negroes are men" it follows that "All honest negroes are honest men," if "All trees are plants" it follows that "All useful trees are useful plants." This process is called *Inference by Added Determinants,* by which from a given proposition we infer another by limiting or qualifying both the subject and the predicate of the original proposition in the same identical manner. Taking the process formally, it is, and must be, always valid. It is as valid as the algebraic process to which it was just now compared. In practice, however, that is, taken materially, the process is often invalid for the reason that the determinant or qualification which is added to subject and predicate, while verbally the same in both cases, is really different, because it takes a different meaning in different contexts. "A cottage is a house"; but it does not follow that "A large cottage is a large house." "A spider is an animal"; but we may not infer "Therefore, a large spider is a large animal." Again, "All bass singers are men," but it would be a false inference to conclude, "Therefore, all good bass singers are good men." The fact is, as is readily seen from these examples, that many adjectives, such as "large," "good," qualify the subject in one way and the predicate in an entirely different way. "Large" is an entirely relative term and "good" has a variety of meanings, such as "artistic excellence," "moral excellence," "physical perfection," etc. *Inference by Complex Conception* is very similar to the process just described. The subject and predicate instead of being determined, or limited, by the addition of a qualifying adjective, may be made parts of a more complex conception, which they determine, or limit. Thus "Poverty is a temptation to crime; therefore, the removal of poverty is the removal of a temptation to crime." "A horse is a quadruped; therefore, the skeleton of a horse is the skeleton of a quadruped." "Arsenic is a poison; therefore, a dose of arsenic is a dose of poison." "Catholics are Christians; therefore, an assembly of Catholics is an assembly of Christians." As in the case of inference by Added Determinants, the process is always valid theoretically speaking. For, if $a=b$, then $x+a=x+b$. In practice, however, care must be taken that the new complex conception does not undergo a change of meaning in conjunction with (1) the subject and (2) the predicate. Thus "Catholics are Christians; therefore a majority of Catholics is a majority of Christians," does not conclude validly, because the term "majority" has a different meaning when applied (1) to Catholics and (2) to Christians.

Rules for Practice. In order to determine the inferences which may be drawn from any given proposition, all that is necessary is (1) to reduce the proposition to its logical form, indicating clearly the subject, copula and predicate, and specifying its quantity and quality; (2) to substitute for subject and predicate some simple symbols, such as S and P, and proceed to convert simply, per accidens, etc.; (3) to restore the original terms by translating the symbols S and P back into the subject and predicate for which they stood. For instance, let the original proposition be "Only Protestant princes can sit on the throne of England." This, reduced to logical form, is

SUBJECT:	COPULA:	PREDIDATE:
All princes who can sit on the throne of England	are	Protestant

This is an A proposition, All S is P. The converse per accidens will be Some P is S, the obverse will be No S is not-P, the contraposit, No not-P is S. The original proposition gives, therefore, as inferences:

(1) Converse per accidens: Some Protestants are princes who can sit on the throne of England.

(2) Obverse: No princes who can sit on the throne of England are not-Protestant.

(3) Contraposit: No not-Protestants are princes who can sit on the throne of England.

When a number of statements relating to the same subject and predicate are to be compared in order to discover the logical relations existing among them, the process is similar to that described in the preceding paragraph. For instance, if we are asked to compare the following propositions:

(1) All crystals are solids
(2) Some solids are not crystals
(3) Some not-crystals are not solids
(4) No crystals are not-solids
(5) Some solids are crystals
(6) Some not-solids are not crystals
(7) All solids are crystals

we should first reduce the propositions to S and P, and then, comparing each pair, we shall find that (2) is the subcontrary of the converse of (1); (3) is inferred from (1) by obverting it, then converting, then obverting once more and finally converting once more. And so for (4) and (1), etc.

CHAPTER XI

Conditional and Disjunctive Propositions

Hypothetical Propositions. In a former chapter[1] we saw that propositions are divided into categorical, conditional, and disjunctive. In treating of Opposition and Conversion we have considered only categorical propositions. Before proceeding to the study of Reasoning, we shall take up the study of conditional and disjunctive propositions.

Conditional Propositions. A conditional proposition is one in which the predication made in one clause is asserted as a consequence of the predication made in the other clause. Thus, "if Caesar was ambitious, he deserved to die," does not assert that Caesar was ambitious, nor that he deserved to die, but that *if* the first predication (He was ambitious) is true, the second (He deserved to die) is true. The first clause in a conditional proposition is called the *Antecedent,* the second is called the *Consequent.* Neither antecedent nor consequent is asserted to be true independently. The truth of the proposition consists in the sequence; if the consequent follows from the antecedent, the proposition is

[1]Chapter VI, p. 84.

true, if it does not follow, the proposition is false. The conjunction "if" is usually a sign of a conditional proposition: "If he is innocent, he should not be punished," "If the barometer falls, it will rain." Occasionally, however, the English idiom allows the use of "if," where there is no real conditionality, as when Mr. Grimwig in *Oliver Twist* declares "If ever that boy returns to this house, sir, I'll eat my head."

MEANING OR IMPORT OF CONDITIONAL PROPOSITIONS. Regarding the import of conditional propositions, it is clear, in the first place, that conditional propositions, if true, imply *some kind of dependence* of the consequent on the antecedent: "If a child is untruthful, he displeases God," here the divine displeasure is asserted to be a consequence, or result, of the child's untruthfulness. In the second place, however, the question arises whether this dependence is always real, or causal. And this is not so easily disposed of as is the first point. In order to state the current solution of the question it is necessary to distinguish between the real, causal, or objective, succession of events, and the notional, rational, or subjective succession of our cognitive states. The former is called the ontological order, the latter the logical order. For example, in the real order, or the ontological order of things, wet feet, a cold, pneumonia, death, is the succession of events, while in the logical order, the order of our knowledge of the event, crape on the door, an inquiry at the house, the answer to the question "What did he die of?" is the succession of our cognitive states in regard to the death of our neighbor. Coming back to the question of the import of conditional propostions, we may safely conclude that the dependence of the consequent on the antecedent is sometimes real, causal, objective, as when we say "If he gets his feet wet he will catch cold," and sometimes the dependence is in the subjective or logical order, as when we say, "If there is crape on the door, some one is dead." The wet feet are the cause, or part of the cause, of the cold. The crape on the door is in no sense a cause of death, but only a sign. This is all very obvious, almost self-evident, and yet it is the answer to a problem which has caused much perplexity to a great many logicians in the present state of philosophy. For if, with the Idealists, we deny that there is a distinction between the subjective and the objective, between thoughts and things, it is not clear how the consequent in a conditional proposition always depends on the antecedent, and yet is not always an effect of the antecedent.

Quantity and Quality of Conditional Propositions. Many writers on Logic deny that distinctions of Quantity and Quality apply to conditional propositions. They assert that all conditional propositions are both singular and affirmative; singular, because they have no extension, and affirmative, because, whether the antecedent and consequent be positive or negative, the proposition always affirms a nexus between them, and in this nexus consists the truth of the proposition.

Thus, "If John does not study his lessons he will not keep up with his class" is an affirmative proposition, although both antecedent and consequent are negative. A few logicians, on the other hand, for the purpose of working out a theory of opposition and conversion of conditionals, introduce distinctions of Quantity and Quality. In this way they obtain the following four types:

If any A is B, C is always D.............. A
If any A is B, C is never D................ E
If any A is B, C is sometimes D............. I
If any A is B, C is sometimes not D........ O

These are, no doubt, schematically and formally correct. As examples may be cited: "If any member of the class is disobedient he will surely be punished" (A); "If any flower is plucked from the stem it cannot grow" (E); "If he aims at the target he may, perhaps, hit it" (I); "If the weather forecast says rain, it may perhaps not rain" (O). It will be evident, however, to any one who examines these statements that they are, in some instances, not genuine conditional assertions. They assert at most a problematic or even a possible dependence of the consequent on the antecedent, whereas a genuine conditional asserts an unquestioned dependence whether in the subjective or in the objective order.

Opposition and Conversion of Conditional Propositions. Those who make the foregoing distinction of Quantity and Quality in conditional propositions, do so for the purpose of applying the rules of opposition and conversion to propositions of this kind. We shall consider merely the case of contradictory and contrary opposition, which alone seems to be of practical value. The conditional statement "If A is B, C is D," is contradicted when we deny that there is a nexus or bond of sequence between the antecedent and the consequent. This may be done by asserting that "C is D does not follow from A is B," or "Even though A is B, it does not follow that C is D." If the original conditional was "If the barometer falls, it will rain," the contradictory would be "Even though the barometer fall, it may not rain," or "The fall of the barometer is not a sign of rain." The latter, is however, a categorical proposition. The former is, consequently, to be preferred as a contradictory of the original conditional. What, then, will be the contrary? The only way in which a contrary can be introduced is by asserting the certainty of the opposite of the consequent. Thus, "If the barometer falls, it will not rain." This is a clear proof that the distinction of opposition among conditional propositions is made possible only by the introduction of modal distinctions, such as "possibly," "necessarily," etc. Applying these modal distinctions we get:

Original.

(A)

If any A is B, C is always D
or necessarily D
or surely D

Contradictory

(O)

If any A is B, C is sometimes not D
or possibly not D
or probably not D

Contrary

(E)

If any A is B, C is never D
or necessarily not D
or cannot be D

Similarly, in the case of the other forms, **E**, **I**, and O, a distinction between the contrary and the contradictory may be made by introducing modal words such as "possibly," "need be," "never can," etc. These are rarely of practical use. Occasionally, however, a form may be fairly natural. For instance, "If a story is believed, it may be true," is an I proposition. Its subcontrary would be "If a story is believed, it need not be true," while the contradictory would be "If a story is believed it cannot be true." In this instance the original proposition may even be converted, and give us (1) "If a story is believed it may not be untrue" (obverse) and (2) "If a story is true, it may be believed" (converse). Again, the proposition "If a country is well governed its people are happy," has for its contradictory "Even if a country is well governed its people need not be happy," and for its contrary, "If a country is well governed its people are not (or cannot be) happy."

Practical Result. The attempt to distinguish the various opposites of conditional propositions does not, as a rule, lead to practical results. One thing, however, which we learn from the attempt may be of practical value. It is, *when we wish to contradict a conditional proposition, all that is required is that we deny the sequence of the consequent from the antecedent;* any attempt to assert the independent truth of the consequent goes farther than contradiction, and tends to establish the contrary of the original conditional.

Disjunctive Propositions. A disjunctive proposition was defined as a proposition which offers two or more alternative assertions or denials. More briefly, it is a proposition which makes an alternative predication. For instance, "All Christians are either Catholics or non-Catholics." Sometimes the number of alternatives is more than two: for example, "Books are either folios or quartos or duodecimos, etc."

IMPORT OF DISJUNCTIVE PROPOSITIONS. The principal question regarding the import of disjunctive propositions is: *Should the alternatives be mutually exclusive, and should the sum of alternatives exhaust all the possibilities?* It is evident that the truth of a disjunctive proposition does not consist in the truth of any one alternative. When I affirm that "It is now either spring, summer, autumn, or winter," I do not affirm that it is spring, nor do I make an independent affirmation about any of the other seasons; all I declare is that it is some one of them. This means that (1) It cannot be both spring and summer, nor can it be any other two seasons at the same time; (2) It must be one of them; or, in other words, there is no other season; the enumeration is complete. In reference to an examination, I may announce the possibilities in the disjunctive assertion "He will either pass or fail or be conditioned," without making a prediction as to which of these contingencies will take place. In both these cases, however, the terms of the alternatives are mutually exclusive, and, whatever my intenton may be, the alternatives cannot be true at the same time, and yet one of them must be true. Take an example of a different kind: "All the students of this class are either industrious or quick to learn." Theoretically speaking, it is possible, and indeed, it often happens, that students are both industrious and quick to learn. But it may happen that in this class none of the industrious students are quick to learn, and all who are quick to learn are deficient in industry. Is the proposition true? The answer is that the proposition "happens" to be true. That is to say, it is true, not by virtue of the form of the proposition, but by reason of the meaning of the terms used. When a proposition merely sums up the result of our observations, the truth may be stated in a disjunctive form; but unless the alternatives are mutually exclusive the proposition is not a true disjunctive. In all true disjunctives the two conditions mentioned above must be verified:

(1) *The alternatives exclude one another.*
(2) *They exhaust all the possibilities.*

The only question remaining is whether, when the alternatives are said to exhaust the possibilities, reference is had to the world of facts or to the world of thought. If I affirm that "All swans are either black or white," the two alternatives are exhaustive of the facts; because, so far as we know, there are no swans that are red or green or blue. If, now, I wish to go beyond the ascertained facts and make a statement which exhausts the possibilities of thought as well as fact I must say, "All swans are either white or not-white." In this case it is absolutely impossible to find a third alternative.

Quality of Disjunctive Propositions. From their very nature, *disjunctive propositions are all affirmative.* They offer a number of alternatives. The negation or exclusion of alternatives would not be disjunctive but categorical; for instance, "Man is neither a plant nor a mineral," is a complex categorical, equivalent to "Man is not a plant" and "Man is not a mineral."

Quantity of Disjunctive Propositions. *Disjunctive propositions are, by their nature, universal,* because they assert that the alternation is necessarily and, therefore, universally valid. Even when the disjunction begins with "some" the enumeration of alternatives is supposed to be universal. For example, "Some men are either criminally negligent or excused through ignorance." Here the reference is to a restricted class of persons; yet it is asserted universally of them that they are either criminally negligent or that, although negligent, they are not criminally so, being excused by ignorance.

Opposition and Conversion of Disjunctive Propositions. Strictly speaking, therefore, and considering the form of the disjunctive proposition, propositions of this kind *do not admit of opposition and conversion.* However, as in the case of conditional propositions, so also here, we may, by introducing modal distinctions, determine the difference between the contrary and the contradictory of a disjunctive proposition. Thus, "A is either B or C," has for its contradictory "A need not be either B or C," and for its contrary "A is neither B nor C." For example, "Every swan is either black or white" has for its contradictory, "A swan need be neither black nor white," and for its contrary, "A swan is neither black nor white." But, these are, obviously, not disjunctive propositions at all: the contradictory is equivalent to "Some swans are neither black nor white" and the contrary is equivalent to "No swan is black," "No swan is white."

Conjunctive Propositions. Besides conditional and disjunctive propositions there is a third class which is sometimes placed parallel with them in the classification of propositions, namely *Conjunctive.* The conjunctive proposition is the opposite of the disjunctive; it asserts the incompatibility of two alternatives. For example, "A man cannot be both innocent and guilty," or, in general "A cannot be both B and C." *The truth of the conjunctive proposition consists in the incompatibility, relative or absolute, of the alternatives. Relative* here means with respect to the same time, place, object or circumstance. For, in the example given, a man may be innocent at one time, in regard to one accusation, and guilty at another time or in regard to another accusation. The conjunctive proposition is always negative.

The terminology adopted in this chapter is somewhat at variance with the general use of the terms, conditional, etc., among modern logicians. It seems most convenient, everything considered, to adopt it. Care should, however, be taken in reading the various authors, to ascertain how their terminology differs from that which is here adopted. It seems best to use the term "Hypothetical" as a genus, and to give a specific meaning to "conditional," "disjunctive," and "conjunctive." The following scheme shows this relation at a glance:

Hypothetical { *Conditional,* "if"
Disjunctive, "either"
Conjunctive, "cannot be both."

CHAPTER XII

The Syllogism

Reasoning. Having treated in the foregoing chapters (1) the logic of terms and mental images, and (2) the logic of propositions and judgments, we now proceed to the *Logic of Reasoning,* which is the third portion of the subject-matter of this science.

Inference has already been described as a process by which the truth contained in a premise or in premises is developed, or evolved, into explicitness in a new proposition, called the conclusion, which follows necessarily from the premise or premises. When there is but one premise, the process is called *Immediate Inference.* When there are several premises, the process is call *Mediate Inference,* or *Reasoning.* When, in the process of Reasoning, we proceed from more general to less general truths the mode of reasoning is *Deductive.* When, on the contrary, we proceed from the less general to the more general, the process is called *Inductive* Reasoning. The type of Deductive Reasoning is the Syllogism.

These terms may be arranged in a scheme of division as follows:

$$\textit{Inference}\begin{cases}\textit{Immediate}\\[2ex] \textit{Mediate, or Reasoning}\begin{cases}\textit{Deductive (Syllogism)}\\ \textit{Inductive}\end{cases}\end{cases}$$

THE SYLLOGISM. The Syllogism may, therefore, be defined as *a process of inference by which from two given propositions we proceed to a third, the truth of which follows from those two as a necessary consequence.* Thus, it being true that "All quadrupeds are animals" and that "All horses are quadrupeds," it follows that "All horses are animals." This is a Syllogism, a process of *deductive reasoning,* which, as is evident, passes from the more general statement about animals and quadrupeds to the less general statement about horses. If from the consideration that "Horses are animals," "Oxen are animals," "Dogs, cats, and other quadrupeds are animals," I proceed to the conclusion "All quadrupeds are animals," I am passing from the less general to the more general, and the process is one of *Induction.* The nature of the Inductive process will be studied in a later chapter. Here we are concerned with the structure and laws of the syllogism.

ANALYSIS OF THE SYLLOGISM. If we take to pieces a typical syllogism, such as,

All quadrupeds are animals;
All horses are quadrupeds;
Therefore, All horses are animals,

we shall readily perceive that there are in it three propositions, the third of which is the *Conclusion* and the other two the *Premises.* Examining the premises and conclusion more closely, we shall observe at once that there are three *Terms,* one of which, "quadrupeds," occurs in both premises. This is called the *Middle Term.* Of the other two, one is the subject of the conclusion; this is called the *Minor Term;* the other is predicate of the conclusion, and is called the *Major Term.* The two premises are called Major and Minor Premise respectively according as they contain the Major or the Minor term.

THE MAJOR, MINOR, AND MIDDLE TERMS. The *Middle Term* is, then, *the term which occurs in both premises* and is not found in the conclusion. The *Minor Term is always the subject of the conclusion,* and the *Major Term is always the predicate of the conclusion.*

MAJOR AND MINOR PREMISES. The *Major Premise* is the premise which contains the *Major Term* and the *Minor Premise* is the premise which contains the Minor Term.

In the example given we may analyse the syllogism into the following component parts:

(1) *Major Premise:* "All quadrupeds are animals."

(2) *Minor Premise:* "All horses are quadrupeds."

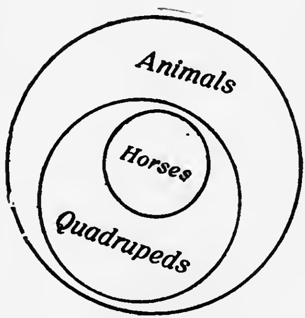
(3) *Conclusion:* "Therefore, All horses are animals."

"Quadrupeds" is the Middle Term, "Horses" is the Minor, and "Animals" the Major Term.

The Vital Thing in the Process. This is an analysis of the structural parts of the syllogistic organism. The vital thing, however, in the syllogism is not mentioned in this enumeration. It is the *conclusive force,* the *consequence,* the *convincing power,* which every valid syllogism has, and which consists in this that the admission of the premises compels the reasoning mind to admit the conclusion also. He who assents to the premises of a valid syllogism is carried on by the nature of the human mind to assent to the conclusion which follows from those premises.

The Cogency of the Syllogism. In trying to account for this cogency on the part of the syllogism, logicians either interpret the premises and read the terms in extension or have recourse to the method of reading by comprehension.

Reading the Proportions in Extension. Let us adopt here, as elsewhere, the extension mode of reading, and look upon the terms as representing classes or groups. The group "quadrupeds" is included in the group "animals"; the group "horses" is included in the group "quadrupeds"; if, now, we try to visualize this relation and represent it by means of circles, we shall see that the group "horses" must necessarily be included in the group "animals" and that this follows as a consequence of the relations which, the premises affirm, exist between "animals," "quadrupeds" and "horses." Thus



If we generalize the case which has just been examined, we find that, in the typical syllogism, there is a larger class represented by the *Major Term*, a smaller class represented by the *Middle Term* and a smallest class represented by the *Minor Term.* The conclusion is affirmed in virtue of the relations between these classes. For "Whatever is affirmed or denied of any class may be affirmed or denied of each member of that class." This is the celebrated *Dictum de Omni et Nullo,* which, according to the majority of authors on logic, is the foundation and justification of all syllogistic reasoning.

READING THE PROPOSITIONS IN COMPREHENSION. Those who prefer to read the propositions of the syllogism in comprehension find that the ground of all syllogistic reasoning is the self-evident principle that "Whatever is a mark of any mark is a mark of that which this last is a mark of." This is known as the *Nota Notæ* and is the counterpart in comprehension of the *Dictum de omni et nullo.* Thus, in the example given, the attribute "animality" is a mark of the attribute "having-four-feet"; this latter is a mark of horses; therefore, "animality" is a mark of horses.

A MORE FUNDAMENTAL PRINCIPLE. Without discussing in detail the relative values of these two principles, we may say at once that they are both derivatives of a fundamental law of thought, namely, the principle which, affirmatively expressed, is called the *Law of Identity* and negatively expressed, the *Law of Contradiction.* The principle of identity asserts that "Everything is itself; A is A; whatever is, is"; the principle of contradiction affirms that "A thing cannot be and not be at the same time; A is not not-A; whatever is not, is not." These are self-evident statements, which, on being applied to the relations existing between the terms in the premises of a syllogism, necessitate the truth of the conclusion.

LAW OF SUBSTITUTION. In mathematics we consider it self-evident that, if two things are *equal* to a third, they are *equal* to each other. This refers to the quantity of things. If we consider the qualities of things, it is equally self-evident that, if two things are *like* a third, they are like each other: if A is like B (in color, or shape, etc.) and B is like C (in color or shape, etc.), then A is like C (in color, or shape, etc.). Now, if we go a step farther, and, instead of considering the quantity or qualities, consider the being, or substance, of things, it is self-evident that, if two things are *identical* with a third, they are identical with each other: if A is identical with, or *is* B, and B *is* C, then A *is* C. If the Maid of Orleans is Joan of Arc, and Joan of Arc is the Shepherdess of Domremy, then the Maid of Orleans is the Shepherdess of Domremy. If all the extension of the term "horses" is identical with some of the extension of "quadrupeds" and all of the extension of "Quadrupeds" is identical with some of the extension of "animals," then all the extension of "horses" is identical with some of the extension of "animals." This principle is sometimes called the LAW OF SUBSTITUTION, and by virtue of it *we may at any time substitute for anything that which is identical with it.*

THE DICTUM AND THE NOTA. For practical purposes the *Dictum* is more convenient than the *Nota,* because, as has been frequently said, we adopt the extensive reading in logic. Besides, the *Dictum* is a more natural statement than the *Nota;* and not, as has been sometimes objected, a tautological assertion. It would be tautological if the group represented by the Major Term were merely an aggregate of individuals and not a true group or class possessing a logical unity of its own.

MILL'S CRITICISM OF THE SYLLOGISM. This last point is made clear by the answer which logicians make to Mill's attack on the validity of the syllogistic process. Mill, looking upon the syllogism as part and parcel of the "medieval" method which he wished to supplant, tried to show that, as an argument to prove the conclusion, the syllogism fails, of its very nature, and cannot conclude validly. The Major Premise, he said, cannot be true unless the conclusion also is true. Therefore, in the Major Premise we assume what we started out to prove, and, consequently, we prove nothing. He takes the following example:

All men are mortal;
The Duke of Wellington is a man;
Therefore, the Duke of Wellington is mortal.

In order, he says, that the Major Premise "All men are mortal" be true, the conclusion "The Duke of Wellington is mortal" must be true. Consequently, if we are honest in saying that "All men are mortal" we must include under "All men" the Duke, whose mortality we intended to prove. This means that we really prove nothing.

When, in answer to this, it was said that the conclusion is, indeed, contained in the Major Premise, but only *implicitly*, Mill replied that the answer proves the truth of his contention that the syllogism is only a trap to catch the unwary, that if in the Major Premise we assert implicitly that the Duke of Wellington is mortal, and do not explicitly include him among "All men," we are not honest with ourselves, or we purposely lay a snare for the person with whom we are arguing.

MILL'S FALSE THEORY OF KNOWLEDGE. The refutation of Mill's criticism of the syllogism deserves to be developed at length because his criticism is founded on *a false conception of the nature of the mind*, a conception held by a great number of logicians to-day, who do not accept the psychology of the Schoolmen. The Empiricists, as they are called, hold that our knowledge is confined to experience, that the mind has no power of adding to concrete experiences by abstraction and other intellectual functions. That the mind has such powers of abstraction and generalization beyond the bounds of actual experience is proved by experience itself.[1]

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[1]Consult Maher, *Psychology*, Chapter XV.

Mill's Criticism Answered. Examining Mill's criticism from the more immediate point of view of logic we remark:

(1) In the process of reasoning we must distinguish between the *truth* of the conclusion and the *knowledge* of the conclusion. It is undoubtedly correct to say that unless the conclusion is true the Major Premise cannot be true: unless it be true that the Duke of Wellington is mortal it cannot be true that "All men are mortal." But a knowledge of the Major Premise does not involve a knowledge of the conclusion; we may know that the Major Premise is true without knowing that the conclusion is true. There are, indeed, propositions which are purely enumerative, as for instance "All the Apostles were Jews", which we cannot know without knowing each and every instance included under the general statement. But, this is a limited class of propositions, and not by any means typical of our process of judgment. All other propositions may be known independently of an examination of particular instances. Now, *Reasoning does not add to truth; it adds to knowledge.* While, therefore, the truth of the Major Premise depends on the truth of the conclusion, the knowledge of the Major Premise does not imply a knowledge of the conclusion. The conclusion adds to our knowledge, and that is all its defenders claim for it.

(2) *But, how do we know the Major Premise?* If the mind had no abstractive power and could merely put together individual instances, of course we could know the Major Premise only by enumerating particular cases, one of which is the conclusion. But, the mind has the power of abstracting universal ideas and building up general principles from a few instances. For the purposes of formal logic we regard the terms of a syllogism as representing groups; we know, however, that it is possible to think of them in comprehension and thus to have a very accurate, explicit knowledge of their meanings, without referring at all to the individuals which they represent.

(3) Mill's example is well chosen for his purpose. Most people understand "All men are mortal" in the extensive sense, and when asked why they believe that all men are mortal, would probably answer: because they have known and learned of so many who died and have never heard (outside a few miraculous exemptions) of a man who lived more than a limited number of years. A physiologist, however, might answer that his *belief in the universal mortality of mankind was based on intrinsic ground and entirely independent of "instances."* He might argue:

All unstable chemical compounds are liable to dissolution; the human body is, therefore, *liable to dissolution, because it is an unstable chemical compound.*

But if, instead of the example which Mill chose, we take the following:

> *All mammals are lung-breathing;*
> *The whale is a mammal;*
> *Therefore, the whale is lung-breathing.*

it is possible on general zoological grounds to hold the Major Premise as true without once thinking of the class "whales." It is possible to think of the attribute connoted by "lung-breathing" as being a characteristic accompaniment in the animal kingdom of the attribute connoted by "mammal." In this, as in all cases, the *truth* of the conclusion is implied in the truth of the Major Premise; but the *knowledge* of the Major Premise does not imply a knowledge of the conclusion.

(4) There are many instances in our daily experience of so-called *inferences which are mere associations,* without any reference to general principles or syllogistic reasoning. "The burnt child dreads the fire" is the summing up of the associations in the child-mind and does not imply that the child reasons:

> *All fire burns;*
> *This is a fire;*
> *Therefore, it burns.*

This and similar processes in the adult or child-mind are capable of being explained without reference to the mind's power of generalizing and abstracting. Other processes, however, which are cases of *genuine syllogistic reasoning* imply the higher mental powers, and only on the supposition that such powers exist can the syllogism be defended and explained.

Comparison, Judgment, Inference. The syllogism is a *process of comparison* in which the Major is compared with the Minor term by means of the identity of each with a common third, the Middle Term. It is a process of *mediate judgment,* because the agreement or difference between the major and the minor terms is pronounced by virtue of the relation which they bear to the middle term. But, above all, the syllogism is an *inference;* our assent to the conclusion is a necessary consequence of our assent to the premises. If the premises are both true and the process is valid, the conclusion must be admitted to be true, because in that case the conclusion cannot be false. However, *we may, by a valid process, derive a true conclusion from false premises.* For instance:

All quadrupeds are vertebrates;
All fishes are quadrupeds;
Therefore, all fishes are vertebrates

is an argument in which the conclusion is true although one premise is false. If the premises are true the conclusion is true; but the inverse is not the case.

Practical Hints. Arguments are seldom expressed in full syllogistic form. Sometimes one premise, and sometimes the conclusion, is suppressed. Thus, "Blessed are the meek; for they shall possess the land." Here we have the conclusion "All meek persons are blessed" and one premise "All meek persons shall possess the land." In order to proceed methodically in the task of supplying the missing premise, it is advisable (1) *to find the conclusion, and in it distinguish the major and minor terms;* (2) *determine the middle term;* and (3) *construct the missing premise.* Examples of this will be given in Chapter XIV.

CHAPTER XIII

The Rules of the Syllogism

Three Terms. From the very nature of syllogistic reasoning it is clear that in every syllogism there must be *three and only three terms.* If there were less than three or more than three terms there could be no comparison, without which there is no mediate judgment. This is the first rule of syllogistic reasoning. Its most important application is in the case when a term, apparently one, is in reality two, because it has two meanings. For instance, in the argument

All criminal actions are punishable by law;
A trial for murder is a criminal action;
Therefore, a trial for murder is punishable by law,

we have in appearance three, but in reality, four terms, because "criminal action" is used in two entirely different meanings.

Three Propositions. From the nature of the syllogism it follows also that there must be in every syllogism *three and only three propositions,* namely the conclusion and two premises. One of these may be suppressed, but should always be capable of being expressed, if need be.

We have then, relating to the structure of the syllogism two Rules:

Rule I. A syllogism should contain three and only three terms.

Rule II. A syllogism should consist of three and only three propositions.

DISTRIBUTION OF THE MIDDLE TERM. Since the *Dictum de Omni et Nullo* applies to every syllogism and is the ground of the validity of the process, and since the Dictum speaks of a class taken in its universality, this class, *the Middle Term, must be taken distributively, at least once.* Besides, unless the Middle Term, the meeting point, so to speak, of the two extremes, be taken distributively at least once, we can have no assurance of the identity of the extremes. Let the middle term be "Americans"; if it is asserted that "All Canadians are Americans" and that "All Brazilians are Americans" there can be no ground for affirming that Brazilians are, or are not, Canadians. Because, although they are both referred to the common third, "Americans," they are not referred to the same part of its extension, and so there is no ground for asserting identity or difference between them. In order to be sure that the comparison of the two extremes is made with the same third, the Middle Term must be distributed in at least one premise. The violation of this rule is called the "Fallacy of the Undistributed Middle." For instance,

All roses are flowers;
All violets are flowers;
Therefore, all violets are roses

is manifestly invalid. The reason of its invalidity is that the Middle Term, "flowers," being the predicate of two affirmative propositions, is in both cases undistributed.

Distribution of the Major and Minor Terms. With regard to the distribution of the Major and Minor Terms, the general rule of all inference holds, namely, that *no term may be taken distributively in the conclusion which was not taken distributively in the premise in which it occurs.* We cannot go beyond our data: we cannot take more out of our premises than we have put into them. We cannot, therefore, take a term more widely in the conclusion than in the premises. When the Major Term is taken more widely in the conclusion than in the Major Premise, the fallacy is called the *Illicit Process of the Major.* For instance,

All diamonds are brittle;
Chalk is not a diamond;
Therefore, chalk is not brittle

is an argument which does not conclude validly. It is guilty of the illicit process of the Major

Term, because "brittle" is distributed in the conclusion, since it is the predicate of a negative proposition; and it is not distributed in the premise, because there it is the predicate of an affirmative proposition. In the argument

No minerals are living substances;
All minerals are solids;
Therefore, no solid is a living substance

the fallacy is an *Illicit Process of the Minor Term,* because "solids," being undistributed in its premise, where it is the predicate of an affirmative proposition, may not be taken distributively in the conclusion.

The Rules, therefore, which refer to the distribution of the Terms, are

Rule III. The Middle Term should be taken distributively in at least one premise.

Rule IV. No term may be distributed in the conclusion which was not distributed in its premise.

Quality of the Propositions. A syllogistic argument is, as has been said, a comparison in which two extremes are brought into relation with a common term, in order to pronounce the agreement or difference between them. For this, it is necessary *that one, at least, of the premises be affirmative.* If both extremes are declared to be unconnected with the Middle

Term, we cannot compare them. From the two negative statements "No bird is a quadruped," "No quadruped is a reptile," it is impossible to determine whether birds are or are not reptiles. "No Frenchman is a Russian," "No Russian is a German," are negative statements which furnish absolutely no ground for affirming or denying the relation between Germans and Frenchmen. Hence the rule: At least one premise must be affirmative, or, as it is usually expressed, from two negative propositions no conclusion can be drawn. Some logicians, indeed, question the absolute validity of this rule; they contend that the following is a valid argument:

What is not metallic is not capable of powerful magnetic influence;
Carbon is not metallic;
Therefore, carbon is not capable of powerful magnetic influence.

Here the two premises are negative, and yet the conclusion seems to be validly drawn. The truth is that if we take the two propositions as negative we have four terms, namely, (1) "What is not metallic," (2) "Capable of powerful magnetic influence," (3) "Carbon," and (4) "Metallic." Either we take "Not-metallic" as one term, in which case there is but one negative premise, or we count "metallic" as one term and "whatever is not metallic" as another, and in that case we have four terms, which prevents the argument from being considered a syllogism at all (Rule I).

If either of the premises is negative, the other premise must be affirmative. In that case the relation of one extreme to the Middle Term is one of disagreement, and the relation of the other extreme to the Middle Term is one of agreement. Between the extremes themselves there must, in every instance of this kind be a relation of disagreement. For when one of two things disagrees with a third and the other agrees with the same third the two cannot agree with each other. This is expressed in the rule: *When one premise is negative the conclusion must be negative.* Thus, the premises "All truthful men are honest" and "Some historians are not honest" can give only a negative conclusion.

There are, therefore, two Rules relating to the Quality of the propositions in a syllogism:

Rule V. From two negatives no conclusion can be drawn.

Rule VI. If one premise is negative the conclusion must be negative.

QUANTITY OF THE PROPOSITIONS. If we consider now the Quantity of the propositions, we may examine in the first place the argument in which both premises are particular. There are only three possible combinations of particular propositions, namely, II, IO, and OO. But (a) OO is not a valid combination, since according to Rule V two negatives give no conclusion. (b) The combination II is also invalid, because, according to Rule III, the Middle Term must be distributed at least once, and the combination II does not distribute any term; both subjects are undistributed because the propositions are particular, and both predicates are undistributed, because the propositions are affirmative. (c) There remains the combination IO, which distributes one term, the predicate of O. But it should distribute two terms, namely the Middle Term (Rule III) and the Major Term, because since one premise is negative the conclusion must be negative (Rule VI) and if the conclusion is negative it must distribute its predicate, the Major Term, and consequently (Rule IV) the Major Term must be distributed in its premise. Therefore, since II, IO, and OO can give us no conclusion, we may formulate the rule: *From two particular premises no conclusion can be drawn.*

There remains a final consideration. If one premise is particular, what is the Quantity of the conclusion? It cannot be Universal. There are four possible combinations of one universal and one particular premise, namely, AI, AO, EI, EO. But (a) EO can give no conclusion (Rule V). (b) AI contains only one distributed term, the subject of A, and that must be the Middle Term (Rule III). If, in this case, the conclusion were universal it should distribute a term (the Minor) which is not distributed in the premise and that would be a violation of Rule IV. (c) In the case of AO and EI there are, indeed, two terms distributed in the premises. But one of these must be the Middle Term (Rule III) and the other the Major Term because the conclusion, being negative, (Rule VI) must distribute the Major, which according to Rule IV must, therefore, be distributed in the premise. Therefore, AO and EI cannot distribute the Minor Term, which means that the conclusion must be particular. The Rule is, consequently, that *When one premise is particular, the conclusion must be particular.*

These two rules, which follow by way of corollaries from the first six rules, relate to the Quantity of the propositions:

Rule VII. From two particular propositions no conclusion can be drawn.

Rule VIII. If one premise is particular the conclusion must be particular.

THE EIGHT RULES. The foregoing rules govern the validity of syllogistic reasoning. These are not arbitrary rules but follow naturally either from the nature of the syllogism itself or from one another. For the purpose of ready reference they are here set down in groups of two:

(A) Structure of the Syllogism:

Rule I. *A syllogism should contain three, and only three, terms.*

Rule II. *A syllogism should consist of three, and only three, propositions.*

(B) Distribution of Terms:

Rule III. *The Middle Term should be taken distributively in at least one premise.*

Rule IV. *No term may be distributed in the conclusion which was not distributed in its premise.*

(C) Quality of the Propositions:

Rule V. *From two negatives no conclusion can be drawn.*

Rule VI. *If one premise is negative the conclusion must be negative.*

(D) Quantity of the Propositions:

Rule VII. *From two particular premises no conclusion can be drawn.*

Rule VIII. *If one premise is particular the conclusion must be particular.*

"Conclusion Follows the Weaker Premise." Rules VI and VIII are sometimes expressed in the saying "*The conclusion always follows the weaker premise.*" The meaning is that a negative is weaker than an affirmative and a particular weaker than a universal. If, then, either premise is negative, the conclusion will agree with that premise and not with the stronger, affirmative, premise; and if either premise is particular, the conclusion will be particular also.

THE COMBINATION IE. As a corollary from the eight rules it follows that "From a particular major premise and a negative minor premise nothing can be inferred." This refers to the combination IE. And the proof is similar to that given in the case of Rules VII and VIII. For, as there is one negative premise the conclusion must be negative. Therefore, it must distribute its predicate, the Major term. Now, the Major premise is I, and cannot distribute either its subject or its predicate. Therefore, if any conclusion were drawn either Rule VI or Rule IV should be violated.

ANALYSIS OF ARGUMENTS. As was pointed out in Chapter XI, syllogistic arguments generally occur in abbreviated form. Sometimes, too, the conclusion is placed either before the premises or between the first and second premises. In analysing arguments and examining their validity the first thing to do is to reduce them to strict syllogistic form, marking off the premises from the conclusion, indicating the minor, major, and middle terms, and then applying as a test of validity the eight rules of the syllogism. The student of logic should not only be able to tell when an argument is not valid, but should also be able to indicate the reason why it is not valid. The following sentence will serve as an example: "How can any one deny that Catholics are unprogressive, since Spain, which is a Catholic country, is notoriously unprogressive?" Here, the conclusion is "No Catholics are progressive." The minor term is, therefore, "Catholics," the major term is "unprogressive," and the middle term must be "Spaniards." The argument in form would be

All Spaniards are unprogressive;
All Spaniards are Catholics;
Therefore, all Catholics are unprogressive.

The formal fallacy of the argument is, of course, an illicit process of the minor term (Rule IV). Besides, if we consider the premises in themselves, it is far from true that "All Spaniards are unprogressive." Everyone who is acquainted with the history of contemporary science, literature, and art can name several Spaniards who are in the van of progress. Moreover, for strict argumentative purposes, it is not true that "All Spaniards are Catholics." However, even if the premises be assumed to be true, the formal invalidity of the argument is apparent as soon as we apply Rule IV.

Epictetus wrote "He is free who lives as he wishes to live. Not one of the bad lives as he wishes; nor is he then free." Here, the conclusion is "No bad person is free." The minor term is "bad person," the major is "free" and the middle term "He who lives as he wishes." The argument in strict form is:

No one who does not live as he wishes is free;
No bad person lives as he wishes;
Therefore, no bad person is free.

This is apparently a violation of the Fifth Rule. On closer examination, however, the argument is seen to consist, not of two negatives, but of one negative (the Major premise) and one affirmative (the Minor), "All bad persons are persons who do not live as they wish." The argument is perfectly valid so far as the rules are concerned. There is, however, an ambiguity in the phrase "lives as he wishes." If a man is consciously and willingly untrue to his ideals, he does not, according to Epictetus, live as he wishes. He may, however, have sufficient knowledge of what he is doing, and sufficient freedom to render his action deliberate and free, in which case only can he be said to be morally "bad." If his conduct is entirely against his wishes, he is not free, of course; but neither is he morally bad, according to the standards of Christian ethics. The argument, therefore, has, in reality, four terms.

CHAPTER XIV

Moods and Figures of Syllogism

THE MOOD OF THE SYLLOGISM. The *Quantity and Quality* of the propositions of a syllogism determine what is called the *Mood* of the syllogism. Thus, the arguments

> *"All the Greeks were lovers of the beautiful; Plato was a Greek; therefore, he was a lover of the beautiful,"*

and

> *"No sensible man scoffs at sacred things; Voltaire scoffed at sacred things; therefore, he was not a sensible man,"*

differ in mood. The former is AAA, the latter is EAE. Since there are four types of proposition namely, A, E, I, and O, and since there are two premises in every syllogism we have *sixteen possible combinations of premises:*

AA, AE, AI, AO;
EA, EE, EI, EO;
IA, IE, II, IO;
OA, OE, OI, OO.

Of these, EE, EO, OE, OO are to be rejected, because from two negative premises no conclusion can be drawn (Rule V). Similarly II, IO, and OI are to be rejected (Rule VII), because from two particular premises no conclusion can be drawn. IE, we saw in the last chapter, is an impossible combination, because it would give an illicit process of the Major Term. There remain

AA,	AE,	AI,	AO
EA,		EI	
IA			
OA			

which, so far as the general rules of syllogistic reasoning are concerned, are valid.

THE FIGURE OF THE SYLLOGISM. The *Figure* of a syllogism is determined by *the position of the Middle Term in the two premises.* Thus, the arguments

> *"All metals are solids; gold is a metal; therefore, it is a solid"*

and

> *"All metals are solids; some metals are heavy; therefore, some heavy (substances) are solids"*

differ in Figure; because, in the first argument the Middle Term, "metal" is subject in the Major and predicate in the Minor premise, while in the second argument the Middle Term, "metal" is subject in both premises. There are only four possible positions which the Middle Term can occupy. It may be (1) Subject in the Major and predicate in the Minor: this is the First Figure. (2) Predicate in both premises: this is the Second Figure. (3) Subject in both premises: this is the Third Figure, or (4) Predicate in the Major and subject in the Minor premise: this is the Fourth Figure. Thus, we have

FIG. I.	FIG. II.	FIG. III.	FIG. IV.
M-P	P-M	M-P	P-M
S-M	S-M	M-S	M-S
——	——	——	——
S-P	S-P	S-P	S-P

Applying the general rules of syllogistic reasoning to each of these figures, we find that the special arrangement of the terms in each necessitates the observance of *Special Rules.*

Special Rules of the First Figure. Here we observe at once that the minor premise must be affirmative; because, if the minor premise were negative the conclusion should be negative (Rule VI). Therefore, P is distributed in the conclusion. Consequently, unless we have an illicit process of the Major, this term must be distributed in its premise. But, there, it is a predicate, and in order to distribute the predicate a premise must be negative. We should, then, have two negatives, from which (Rule V) no conclusion can be drawn.

We observe, in the next place, that the major premise must be universal. The Minor, being affirmative, does not distribute its predicate, the middle term; but the middle term must be distributed once (Rule III). Therefore, the premise, which has M for its subject, must be universal.

We have, therefore, two special rules for the first figure.

I. *The minor premise must be affirmative.*
II. *The major premise must be universal.*

Applying these rules to the eight universally valid moods given at the end of the first paragraph of this chapter, AE and AO are rejected by Special Rule I, and IA and OA by Special Rule II. We have left, as valid in the first figure

AAA, EAE, AII, and EIO

Special Rules of the Second Figure. Here the middle term is predicate in both premises. It follows that one premise must be negative; otherwise M would not be distributed, as it is required to be by General Rule III. But, since one premise is negative, the conclusion must be negative. Therefore, the major term, predicate of the conclusion, must be distributed in the major premise, which necessitates the major premise being universal.

The special rules for the second figure are, therefore,

Rule I. One premise must be negative.

Rule II. The major premise must be universal.

Applying these rules to the eight universally valid moods we exclude by Rule I, AA, AI, IA and by Rule II, IA and OA. We have left as valid in the Second Figure

EAE, AEE, EIO, and AOO.

Special Rules of the Third Figure. In this Figure the middle term is subject in both premises. As in Figure I, so here also, the minor premise must be affirmative; because a negative Minor would involve a negative Major, and we should have two negative premises. Now, since the Minor must be affirmative, S is not distributed in its premise; therefore, it may not be distributed in the conclusion (General Rule IV), which means that the conclusion must be particular.

The Special Rules for the third figure are, therefore:

Rule I. The minor premise must be affirmative.

Rule II. The conclusion must be particular.

If we apply these rules to the eight universally valid moods we are obliged to exclude AE and AO. There are left AAI, IAI, AII, EAO, OAO and EIO.

Special Rules of the Fourth Figure. In this Figure the middle term is predicate in the major and subject in the minor premise. The rules for this figure are given in conditional form.

(1) *If the major is affirmative, the minor must be universal.* Otherwise, the middle term would not be distributed.

(2) *If the minor is affirmative the conclusion must be particular.* If it were universal, we should have an illicit process of the minor term.

(3) *If the conclusion is negative the major must be universal.* Otherwise we should have an illicit process of the major term.

The application of these rules excludes AI (Rule I), AO (Rule I), and OA (Rule III). We have left

AAI, AEE, IAI, EAO, and EIO.

Nineteen Valid Moods. We have, therefore, in all, nineteen valid moods, four in the first figure, four in the second, six in the third and five in the fourth.

Mnemonic Lines. In order to remember these, the medieval writers invented a set of memory verses, in which the vowels of the words bArbArA, etc., stand for the propositions of the syllogism. The following is a modification of the lines invented in the thirteenth century by William of Shyreswood and first given wide circulation in the same century by Peter the Spaniard, afterwards Pope John XXI.

Barbara, Celarent, Darii, Ferioque, *prioris;*
Cesare, Camestres, Festino, Baroco, *secundae;*
Tertia Darapti, Disamis, Datisi, Felapton,
Bocardo, Ferison *habet; quarta insuper addit*
Bramantip, Camenes, Dimaris, Fesapo, Fresison.

In the next chapter we shall see that almost every consonant as well as the vowels in these strange-sounding words has a meaning. The vowels mean the propositions, A, E, I, O, which make up the valid syllogisms in each figure. The lines have been said to contain more meaning within a limited space than any other lines of similar length that have ever been penned.

COMPARISON OF THE FOUR FIGURES. Comparing now the four figures, we recognize the preeminence of the first, or, as it is sometimes called, the perfect, figure.

(1) In it we can prove either of the four kinds of propositions, A, E, I and O.

(2) It is the only figure in which we can prove an A proposition.

(3) It is the only figure in which the subject of the conclusion is subject in its premise and the predicate of the conclusion, predicate in its premise.

(4) To it alone can the *Dictum de omni et nullo* be applied directly.

The second figure lends itself naturally to disproof, rather than to proof. All its conclusions are negative.

The third figure is the most convenient figure for proving exceptions. All its conclusions are particular.

The fourth figure reverses in its premises the order which exists in the first figure. For that reason it was not admitted by Aristotle, and is rejected by many logicians down to the present time. It was introduced by Galen, a physician, who lived in the second century of our era, and it is sometimes called the *Galenian Figure.* Its practical importance is very slight. Symmetry, however, demands that it be retained as a possible, though it is seldom an actual, or spontaneous, mode of reasoning.

Those who reject the fourth figure maintain that all its conclusions are obtainable in the first figure, by a process of conversion. The moods *Bramantip,* etc., are, therefore, called *Indirect Moods* of the first figure. For example, the following argument may be stated in the fourth figure as *Bramantip:*

> *All violets are flowers;*
> *All flowers are plants;*
> *Therefore, some plants are violets.*

If we take the same premises, in inverse order, thus

All flowers are plants
All violets are flowers

we get the conclusion in the first figure, "Therefore, all violets are plants," and by converting this *per accidens* we get the original conclusion "Therefore, some plants are violets." Thus *Bramantip* is said to be an indirect mood of *Barbara*. Similarly, in the case of the other moods of the Fourth Figure. Take an instance of *Fresison:*

No rose is a violet;
Some violets are fragrant flowers;
Therefore, some fragrant flowers are not roses.

By converting both premises simply we may argue in *Ferio* of the First Figure:

No violet is a rose;
Some fragrant flowers are violets;
Therefore, some fragrant flowers are not roses.

Practice. In order to become thoroughly conversant with the different syllogistic forms it is advisable for the student of logic to practice the art of (1) constructing syllogisms to exemplify the various moods and figures, and of

(2) analysing given arguments and determining the mood and figure to which they belong.

(1) Suppose we are asked to construct a syllogism in *Baroco*. We know that *Baroco* is in the second figure. Therefore, the position of the middle term will be predicate in both premises. The skeleton, so to speak, of the structure will be

$$\begin{array}{cc} P & M \\ S & M \\ \hline S & P \end{array}$$

We know also that the mood is AOO. The major will be a universal affirmative, the minor a particular negative, and the conclusion a particular negative. We may, therefore, write

All P is M;
Some S is not M;
Therefore, some S is not P.

The next step is to supply the material of the argument, to fix on some terms in place of P, M, and S. Let the major term be "wood," the middle term "combustible" and the minor term "chair." The argument in *Baroco* will be:

All wood is combustible;
Some chairs are not combustible;
Therefore, some chairs are not wood.

(2) Let the argument which is offered for analysis be: "It cannot be maintained that all

the Latins are unscientific; because everyone knows that some of the Italians have cultivated to a high degree the spirit of scientific inquiry." Here the aim is to prove that "Some Latins are successful scientists." This is the conclusion. The minor term is, therefore, "Latins"—the minor is always the subject of the conclusion. The major term is "successful scientist"—the major term is always the predicate of the conclusion. The middle term is "Italians." Now, two assertions are made about the middle term: (a) It is expressly stated that "Some Italians are successful scientists" and (b) it is implied that "All Italians are Latins." The argument, therefore, is,

Some Italians are successful scientists;
All Italians are Latins;
Therefore, some Latins are successful scientists.

From the position of the middle term it is evident that the argument is in the Third Figure. The mood, as is seen from the quantity and quality of the propositions, is IAI. IAI in the third figure is a valid form, *Disamis*.

REMARKS. Looking over the conclusions in the four figures, we remark that (1) A can be proved in only one mood, and only in the First Figure. (2) E can be proved in four moods and in every Figure except the third. (3) I can be proved in six moods, and in every Figure except the second. (4) O can be proved in eight moods and in every Figure. From these observations we infer that *A is the most difficult, and O the easiest, proposition to prove.* A universal affirmative statement is of the greatest value in science, and is, in debate, a much more pretentious statement than a negative or a particular. But it should be advanced with great caution, because when we attempt to prove it, we have at our disposal only one mood and one figure, namely *Barbara,* AAA in the first figure. On the other hand, our opponent in debate who wishes, naturally, to disprove the A proposition which we have advanced, has only to prove the contradictory, O, and for his purpose he can avail himself of any of eight moods, and can use any of the four figures. Hence the advice of the ancient writers on logic: Avoid universal statements; *Latet dolus in generalibus,* there are pitfalls in the path of him who indulges in general assertions.

CHAPTER XV

Reduction of Syllogisms, Enthymeme, etc.

Reduction. The first figure being the perfect figure and the only figure to which the *Dictum de omni et nullo* can be applied directly, it becomes necessary to reduce the moods of the second, third, and fourth figures to the first if we wish to test their validity. The process by which a mood in the second, third, and fourth figures is brought back to the first is called *Reduction.* Some logicians, it is true, do not consider that reduction to the first figure is the only proof of validity. Nevertheless, they regard reduction as a useful process because it "makes evident the essential unity of all forms of syllogistic inference, and systematises the theory of syllogism by showing that all the various moods are, at bottom, expressions of but one principle."*

Meaning of Words Barbara, etc. The words *Barbara,* etc., in the mnemonic lines given in Chapter XIV contain all the rules according to which the process of reduction is performed:

(1) The initial consonants are the four first consonants of the alphabet, B, C, D, and F. They

*Welton, *Manual of Logic,* I, 353.

indicate the mood in the first figure to which the mood in question is to be reduced. If we undertake to reduce *Dimaris,* we know at once that it is to be reduced to *Darii; Camenes* is reduced to *Celarent, Ferison* to *Ferio, Baroco* to *Barbara,* etc.

(2) The letter *s* in the body of a word indicates that the proposition signified by the vowel preceding it is to be converted simply.

(3) The letter *p* in the body of a word indicates that the proposition signified by the vowel preceding it is to be converted per accidens.

(4) The letter *m* in the body of a word indicates that the order of the propositions is to be inverted, that which was major becoming the minor, and that which was minor becoming the major premise in the new syllogism.

(5) The letter *c* in the body of a word indicates that the process of conversion is to be indirect or, as the mathematicians say, *per impossible.*

Examples of Reduction. The application of these rules may be illustrated by the following examples:

(A) *Cesare* to *Celarent.* In the "Ethics" Aristotle argues "The affections are not acts of choice. The virtues are acts of choice; therefore, the virtues are not affections." This may be cast in the mood *Cesare:*

No affection is an act of choice;
All virtues are acts of choice;
Therefore, no virtue is an affection.

The letter *s* following the first E of *Cesare* indicates that the major premise is to be converted simply. We have, therefore,

No act of choice is an affection;
All virtues are acts of choice;
Therefore, no virtue is an affection,

which is a syllogism in *Celarent*, proving the same conclusion as *Cesare* from the same premises. Thus *Cesare* is reduced to *Celarent.*

(B) *Disamis* to *Darii.* The argument implied in the sentence, "Some words in English, pronouns for example, are inflected, because they are of Anglo-Saxon origin," may be cast in the mood *Disamis:*

Some pronouns are inflected;
All pronouns are of Anglo-Saxon origin;
Therefore, some words of Anglo-Saxon origin are inflected.

The *m* in *Disamis* indicates a transposition of the premises; the first *s* indicates the simple conversion of the major premise and the last *s* indicates the simple conversion of the new conclusion. Thus, we have

All pronouns are of Anglo-Saxon origin;
Some inflected words are pronouns;

Therefore, some inflected words are of Anglo-Saxon origin,

a syllogism in *Darii*. If, now, we convert the conclusion simply, we have "Some words of Anglo-Saxon origin are inflected," which is the conclusion of the original *Disamis*.

(C) *Bramantip* to *Barbara*. The following is an argument in *Bramantip*

All poets are imaginative;
All imaginative persons are sensitive;
Therefore, some sensitive persons are poets.

The *m* in *Bramantip* tells us that the premises must be transposed. The final letter, *p*, indicates that the new conclusion must be converted per accidens. Thus we obtain a syllogism in *Barbara:*

All imaginative persons are sensitive;
All poets are imaginative;
Therefore, all poets are sensitive;

which, by conversion *per accidens* of the conclusion, gives

Some sensitive persons are poets.

(D) *Baroco* and *Bocardo* (they contain *c* in the body of the word), cannot be reduced directly, the premise *O* being a proposition which resists both simple conversion and conversion per accidens. They are reduced by the indirect process. That is to say, we assume for the sake of argument, that the premises are true, but the conclusion false, and then show that from that assumption some absurd consequence follows.

Take an example of *Baroco:*

All good actions are praiseworthy;
Some acts of pious persons are not praiseworthy;
Therefore, some acts of pious persons are not good.

Let us suppose that both the premises are true, but that the conclusion is false. If the conclusion is false, its contradictory is true, namely, "*All acts of pious persons are good.*" Combining this with the original major premise, we have a syllogism in *Barbara:*

All good actions are praiseworthy;
All acts of pious persons are good;
Therefore, all acts of pious persons are praiseworthy.

But, this is incompatible with the original minor which we supposed to be true. Therefore, if we suppose that the premises of *Baroco* are true and the conclusion false, an absurdity follows. Therefore, if the premises are true, the conclusion is true, which means that *Baroco* is valid.

An example in *Bocardo:*

Some witches were not morally responsible;
All witches were put to death;
Therefore, some persons who were put to death were not morally responsible.

Here, again, let us suppose that the premises are true and the conclusion false. If the conclusion is false, its contradictory, "*All persons who were put to death were morally responsible,*" must be true. If we combine this with the original minor premise we have

All who were put to death were morally responsible;
All witches were put to death;
Therefore, all witches were morally responsible,

which is incompatible with the major "*Some witches were not morally responsible.*" Therefore, if we suppose the conclusion of *Bocardo* to be false we suppose the major premise to be false and true at the same time, which, as we say in geometry, is absurd.

Baroco and *Bocardo* are the only forms which require the indirect method of reduction. All others may be reduced by the direct method, and although they may be reduced indirectly also, the direct method is preferable, because of its greater simplicity and the greater facility with which it is used.

THE ENTHYMEME. A syllogism, as we have seen, is seldom expressed in its complete form except in the most formal discourse or written argumentation. Usually one premise or the conclusion is understood, that is, implied, and not expressed in language. An argument which suppresses one of the premises or the conclusion is called an *Enthymeme.* The sentences of the Sermon on the Mount are examples of this form of expression. "Blessed are the clean of heart; for they shall see God," is an enthymeme, containing the conclusion and one of the premises of a syllogism. In full syllogistic form it would be:

All who shall see God are blessed;
The clean of heart shall see God;
Therefore, they are blessed.

Whenever the words "because," "since," "for," etc., which are called illative particles, because they indicate an inference, occur, we may look for an enthymeme. For instance, "The British government is constantly changing its policy, because it is democratic." There is often a fallacy in such arguments, and the surest method of detecting the hidden flaw or weakness of the argument is to proceed at once to the formulation of the missing premise. In the instance given, the missing premise is the major, "All democratic governments are constantly changing their policy." Frequently, the general principle implied needs but to be formulated in order to be refuted. "He has no appreciation of poetry, for he is a mathematician" implies, if it is a valid argument, that "No mathematician has an appreciation of poetry," which is far from being true.

POLYSYLLOGISM, CHAIN OF REASONING. By making the conclusion of one syllogism the premise of another syllogism we may construct what is known as a *Chain of Reasoning*, or *Polysyllogism*. For instance,

(1) No simple substances can be dissolved into parts;
(2) All spiritual substances are simple;
(3) Therefore, no spiritual substance can be dissolved into parts;
(4) The soul is a spiritual substance;
(5) Therefore, it cannot be dissolved;
(6) What cannot be dissolved is immortal;
(7) Therefore, the soul is immortal.

Here 1,2, and 3 form a simple syllogism; 3,4, and 5, and 5, 6, and 7 also form syllogisms, but since one premise in each case is the conclusion of a previous syllogism the argument has a special name, *Episyllogism;* and the first, consisting of 1, 2, 3, is called a *Prosyllogism,* that is, a syllogism the conclusion of which forms a premise in a subsequent syllogism. The whole series is a chain of reasoning.

The Sorites. Another variety of chain of reasoning is that in which the series is abridged by suppressing the conclusions of the prosyllogisms. For example, Aristotle in the "Poetics" argues "Action is that in which happiness lies; what contains happiness is the end and aim; the end and aim is what is highest; therefore, action is what is highest." This form of chain reasoning is known as the *Sorites,* a word which has reference to the Greek word σωρός, a heap. The Sorites is of two kinds, one is known as the Aristotelian, the other as the Goclenian. The Aristotelian form is:

All A is B
All B is C
All C is D
Therefore, All A is D.

The Goclenian form is:

All C is D
All B is C
All A is B
Therefore, All A is D.

The Aristotelian Sorites is, then, a series of propositions so arranged that the predicate of the first becomes the subject of the second, the predicate of the second, the subject of the third, and so on, until finally the subject of the first and the predicate of the last premise are united to form the conclusion.

Rules of the Sorites. It is not difficult to deduce from the general rules of syllogistic reasoning special rules for the Aristotelian Sorites. It is evident, in the first place, that only one premise can be negative; for if two premises were negative we should somewhere in the series have a combination of two negative premises, from which, as we know, no conclusion may be drawn. In the next place, it is clear that if any premise is negative, it must be the last: for, if any premise is negative the conclusion must be negative, that is to say, the term D is distributed in the conclusion. It must, therefore, be distributed in the premise, which necessitates that premise being negative. Finally, for similar reasons, we cannot have more than one particular premise, and if any premise is particular, it must be the first. Because, the middle term must be distributed at least once: if we had two particulars, or if any premise but the first were particular, there should be one syllogism which would not distribute the middle term. The rules for the Aristotelian Sorites are, therefore:

Rule I. There can be only one negative premise.

Rule II. If any premise is negative, it must be the last.

Rule III. There can be only one particular premise.

Rule IV. If any premise be particular, it must be the first.

THE EPICHIREMA. Allied to these, as a species of chain reasoning, is the form called the *Epichirema,* which is a syllogism containing, besides the two premises, the proof or part of the proof of one of the premises. The example commonly given is "Whatever is spiritual is immortal, because it is incapable of corruption. The human soul is spiritual. Therefore, it is immortal." This argument is the syllogism:

Whatever is spiritual is immortal;
The human soul is spiritual;
Therefore, it is immortal,

to which is added part of the proof of the major premise. The proof of the major premise is:

Whatever is incapable of corruption is immortal;
The spiritual is incorruptible;
Therefore, it is immortal.

HINTS FOR PRACTICE. The distinction of the Epichirema from the Prosyllogism and the Episyllogism has very little to do with the laws of correct and valid reasoning. It is a distinction which refers more to the expression of the reasoning process than to the value of that process in itself. Consequently, it is more important for the rhetorician than for the logician. So far as the logician is concerned the only point of importance is this: it is easy to introduce an unproved or an untrue premise in a chain of reasoning, and the abbreviated form of the Enthymeme, Prosyllogism, Episyllogism, Sorites, and Epichirema lends itself to the art of deception. To test these processes, therefore, it is well to expand the argument into an explicit series in which all the premises and all the conclusions are expressed. Montaigne argued "If God exists, He possesses life; if He has life, He has senses; if He has senses, He is subject to corruption." Taking this out of the conditional form, the argument would be "Whatever exists possesses life; God exists; therefore, He possesses life. Whatever possesses life has senses; God possesses life; therefore, He has senses." We may stop here, for, whatever may be said about the first general statement, "Whatever exists possesses life," which is not true, the second general statement, "Whatever possesses life has senses" is manifestly false—since there is such a thing as intellectual and spiritual life which does not imply the possession of bodily senses.

CHAPTER XVI

Hypothetical Reasoning, The Dilemma, Etc.

Pure and Mixed Disjunctive Reasoning. The syllogism, as it has been considered so far, is made up of categorical propositions, and is called the categorical syllogism. There is, besides the categorical form, a hypothetical form of reasoning, the propositions of which are either all hypothetical or partly hypothetical and partly categorical.

The *purely hypothetical syllogism,* which is made up of hypothetical propositions, need not detain us here. It is a very unusual form of reasoning, and, since, as we saw in Chapter XI, hypothetical propositions do not, strictly speaking, admit of distinctions of quantity and quality, the purely hypothetical syllogism has no moods or figures.

The *Mixed Syllogism* is one in which some of the propositions are hypothetical and some categorical. It has two varieties, the *Mixed Conditional* and the *Mixed Disjunctive* (see end of Chapter XI).

Mixed Conditional Syllogism. The *Mixed Conditional Syllogism* has for its major premise a conditional proposition and for its minor premise a categorical. For example,

If he is industrious he will succeed;
He is industrious;
Therefore, he will succeed.

We learned in Chapter XI to distinguish the two portions of the conditional proposition and call them *antecedent* and *consequent*. In the example given, the minor premise affirms the antecedent, and the conclusion affirms the consequent.

MOODS OF THE CONDITIONAL SYLLOGISM. From the nature of the relation between antecedent and consequent, it follows that

(1) From the affirmation of the antecedent we may proceed inferentially to the affirmation of the consequent. The meaning, or import, of conditional propositions is that the consequent follows from the antecedent. "If A is B, C is D." When, therefore, in the minor we assert categorically that "A is B," it follows that "C is D." This is the positive mood of the conditional syllogism, and is known as the *Modus Ponens*.

(2) From the negation of the consequent, we may proceed inferentially to the negation of the antecedent. For, if the consequent be not true as a categorical statement, it must be that the antecedent is not categorically true. "If A is B, C is D," is the major; the minor is "C is

not D'' and the conclusion, "Therefore, A is not B.'' This is the negative mood of the conditional syllogism, and is called the *Modus Tollens*.

(3) We may not proceed from a denial of the antecedent to a denial of the consequent. For, although the consequent follows from the antecedent, it may follow from other antecedents also, and the denial of one antecedent, does not warrant us in denying the consequent. Let us suppose that the major premise is, *If Caesar was ambitious he deserved to die,* we may not argue *He was not ambitious, therefore, he did not deserve to die,''* because he may have deserved to die on some other count, murder, treason, etc., besides ambition.

(4) Neither is it allowed from an affirmation of the consequent to infer an affirmation of the antecedent. We may not, for example, argue *If Caesar was ambitious, he deserved to die: Caesar deserved to die; therefore, he was ambitious.''* For he might deserve death on account of treason, murder, or some other crime.

Rules of Conditional Reasoning. We may sum up the rules for conditional reasoning in one principle: *In affirmation it is allowed to proceed from the antecedent to the consequent; in negation it is allowed to proceed from the consequent to the antecedent.*

Mixed Disjunctive Syllogism. The *Mixed Disjunctive Syllogism* is an argument in which the major premise is a disjunctive and the minor a categorical proposition. In the disjunctive major several alternatives are proposed. The minor affirms or denies one or more of these alternatives, and the conclusion affirms or denies the remaining alternatives. The following is a type of disjunctive reasoning:

Either A is B or C is D
A is B
Therefore, C is not D.

The principle on which disjunctive reasoning is based is: The denial of one alternative justifies the affirmation of the other, or of the others; the assertion of one alternative justifies the denial of the other, or of the others. This, of course, assumes that the enumeration of alternatives is complete and that the alternatives are mutually exclusive.

He is either honest or dishonest
He is not dishonest
Therefore, he is honest

is a case in which the conditions are fulfilled in an ideal way. In practice, however, it is not always easy to be certain either that the enumeration of alternatives is complete or that the alternatives are mutually exclusive.

He is either a fool or a knave
He is a fool
Therefore, he is not a knave

errs in both respects. The major should be *He is either a fool or a knave, or both, or neither,* because one may be both a fool and a knave at once, and there are many persons who are both sensible and honest. From the fact, therefore, that he is a fool it does not follow that he is not a knave.

RULES OF DISJUNCTIVE REASONING. The rules, therefore, for disjunctive reasoning are:

I. The enumeration of alternatives must be complete.
II. The alternatives must be mutually exclusive.

MOODS OF MIXED DISJUNCTIVE SYLLOGISM. There are two recognized moods of the disjunctive syllogism. We may (a) posit, that is, affirm, one alternative in the minor premise and sublate, or deny, the other alternatives in the conclusion. This is called the *Modus Ponendo Tollens.* An example would be:

Either the soul is immortal or death ends all things
The soul is immortal
Therefore, death does not end all things.

We may (b) sublate, that is, deny, one alternative in the minor premise and posit, or affirm, the others in the conclusion. This is called the *Modus Tollendo Ponens.* Thus,

Either the soul is immortal or death ends all things
But death does not end all things
Therefore, the soul is immortal.

It should be remarked that when the major premise enumerates more than two alternatives and the minor affirms or denies only one, the conclusion must deny or affirm the others disjunctively, not categorically. For example

A is either B or C or D
A is not B
Therefore, A is either C or D.

He is either a Frenchman, an Italian or a Spaniard
He is not a Frenchman
Therefore, he is either an Italian or a Spaniard.

PRACTICAL DIFFICULTY. The practical difficulty in using the disjunctive form of reasoning arises from the difficulty, amounting sometimes to impossibility, of ascertaining whether the enumeration of alternatives is complete and whether the alternatives are mutually exclusive.

The Dilemma. Both the conditional and the disjunctive forms of reasoning are combined in the very popular species of argument known as the *Dilemma.* The name arises from the fact that in this mode of argument the adversary is offered two alternatives, either of which is shown to be damaging to his side of the question. There is, however, no reason why the alternatives should be merely two. A trilemma or a quadrilemma, offering three and four alternatives respectively, would be just as valid. In the traditional form, there are only two alternatives, the two horns, as they are called, of the dilemma. The peculiar force of the argument is due to the inconvenience to which the adversary is subjected in being compelled to choose between two alternatives, or rather in being shown that whichever of the two he chooses he will find himself regretting that he did so.

Definition of a Dilemma. The dilemma may be defined as a syllogism in which the major premise is a compound conditional proposition and the minor a disjunctive containing two alternatives each of which is disagreeable and damaging to the adversary. When the disjunctive affirms the truth of one of the antecedents, the dilemma is positive, or *Constructive;* when it denies one of the consequents, the dilemma is negative or *Destructive.* Both the constructive and the destructive dilemma may be either *Simple* or *Complex.* We have, consequently, four kinds of dilemma:

(A) SIMPLE CONSTRUCTIVE,

If A is B, E is F; and if C is D, E is F
Either A is B or C is D
Therefore, E is F.

If he steers towards Scylla he will be wrecked; and if he steers towards Charybdis, he will be wrecked. But, he must steer either towards Scylla or towards Charybdis. Therefore he will be wrecked.

(B) COMPLEX CONSTRUCTIVE,

If A is B, E is F; if C is D, G is H,
Either A is B or C is D
Therefore, either E is F or G is H.

"If the Christians have committed crimes your refusal to grant an inquiry is irrational; if they have not, your refusal is unjust. But, either the Christians have committed crimes or they have not. Therefore, your refusal is either irrational or unjust." (From Tertullian's argument against the policy of Marcus Aurelius.)

(C) SIMPLE DESTRUCTIVE,

If A is B, E is F, and if A is B, G is H
Either E is not F or G is not H
Therefore, A is not B.

If he is to sail through the straits unharmed he must avoid Scylla and Charybdis. But either he will not avoid Charybdis or he will not avoid Scylla. Therefore, he will not sail unharmed.

(d) Complex Destructive,

If A is B, E is F, and if C is D, G is H
Either E is not F or G is not H
Therefore, either A is not B or C is not D.

"If he were reverent he would not speak disrespectfully of the Scriptures in earnest; if he were wise he would not speak disrespectfully of them in jest. But either he speaks in earnest or in jest. Therefore, either he is not reverent or he is not wise."

The Strength and Weakness of the Dilemma. The dilemma is, of its nature, a very telling argument. Its strength, however, is rhetorical rather than logical. It always puts the adversary in an unpleasant position, and whether he extricate himself or not, the advantage, for the time being at least, is on the side of the person who has succeeded in putting him there. The mere fact that the dilemma supposes an adversary, and that its success consists in inconveniencing the adversary for the time being proves that it is mainly a rhetorical, not a logical, device. From the point of view of logic, therefore, the dilemma is a weak form of argument.

Rules of the Dilemma. In order that a dilemma conclude logically it must conform to certain requirements:

I. *The enumeration of alternatives in the minor must be complete.* This condition is very rarely fulfilled. Socrates, for example, argued *If death ends all things, it is not to be feared; if death is the beginning of a happy life with the gods and heroes in Elysium, it is not to be feared. But either death ends all things or it is the beginning of a life in Elysium. Therefore, it is not to be feared.* Socrates omitted the third alternative, the possibility of death being the beginning of an unhappy existence, which, apart from the irrational, instinctive, dread of death, is the real reason why death is sometimes feared.

II. In the major premise, care should be taken that the consequents drawn from the antecedents, and no others, really follow from the antecedents. If a consequent is drawn which does not really follow, or if a consequent which really follows is omitted, the argument is not valid. It is related of the Caliph Omar, who is said to have burned the Alexandrian Library in 640 A. D., that he met the remonstrances of the custodians of the Library with the dilemma: *"If your books are in conformity with the Koran, they are superfluous; if they are not, they are pernicious. But, either they are in conformity*

[redacted]

with the Koran or they are not. Therefore, either they are superfluous or they are pernicious." The minor, of course, is hardly a complete enumeration of the alternatives in the case of books which dealt with mathematics or pure science. But, passing over that defect, we remark that the inferences drawn in the major are not conclusive. It does not follow that books which agree with the Koran are superfluous or useless. They may be very useful and their usefulness may be due to the fact that they elucidate, explain, or illustrate the doctrines of the Koran.

III. *A dilemma must be such that it cannot be rebutted.* A dilemma is rebutted when the adversary can take the same alternatives, draw consequents opposed to the original consequents, and so establish a conclusion the very opposite of the original conclusion. Rebuttal shows the logical weakness of the dilemma rebutted, and at the same time not only robs it of all rhetorical force, but throws the advantage very decidedly in favor of the person who rebuts the original argument. The man who frames a dilemma that is liable to rebuttal places in the hands of his adversary a ready-made weapon that can be used with telling effect. The following are historical instances of dilemmas cleverly rebutted:

(a) Protagoras, the sophist, bargained to teach Eualthus law on condition that the fee should be paid when the pupil won his first case. Eualthus, having finished his course, took up some other business, refusing to undertake a suit. Whereupon Protagoras brought suit against him for the promised fee and Eualthus defended the suit. Protagoras argued, *O judges, this young man must pay me my fee; for if he lose this suit, he must pay me by order of the court, and if he win this suit he must pay me according to the terms of our contract.* To this Eualthus answered, *O judges, I am not bound to pay this sapient master. For, if I lose the suit, I am not yet bound by the terms of the contract, and if I win the suit, I am absolved by the decision of the court.* The judges, we are told, left the case undecided.

(b) An Athenian mother, wishing to deter her son from entering public life, argued, *If you act justly you will displease your fellowmen, and if you act unjustly, you will displease the gods. But, you must act either justly or unjustly. Therefore, either you will displease men or you will displease the gods.* The son—a pupil of the sophists, perhaps—answered, *If I act justly the gods will love me, if I act unjustly men will love me. But I must act either justly or unjustly. Therefore, if I enter public life, I shall be beloved either by the gods or by my fellowmen.*

Practical Hint. Because of the difficulty of observing these conditions in practice, the dilemma, while theoretically a valid argument, is more often fallacious than valid. In scientific exposition and demonstration it should be used rarely, and with the greatest caution.

CHAPTER XVII

Induction

Two Kinds of Reasoning. In Chapter XII we distinguished two kinds of Reasoning, namely, *Deduction,* which proceeds from the more general to the less general, and *Induction,* which proceeds from the less general to the more general. We must now take up the study of the Inductive process and determine more in detail its nature and scope.

Inductive Reasoning. All arguments in which we start with particular facts and arrive at a conclusion which is a general statement or a universal law are inductive in their nature. Sometimes, as in geometry, we use a particular instance, for example, the triangle A B C, merely to bring the abstract qualities vividly before our minds. We argue from general qualities to other general qualities, not from the particular instance. This is not induction at all, but deduction. Sometimes we take particular instances and, having enumerated them completely, we infer a general statement regarding the whole class to which they belong. Thus, we observe that cows chew the cud, and that sheep and goats chew the cud. We next observe that cows, sheep and goats are horned animals. From which we are warranted in summing up our observations by saying that "All horned animals that we have observed are ruminants." This is induction by the *Enumeration,* or accumulation, of instances. When the enumeration is complete, the process is called *Perfect Induction;* when it is incomplete, the process is called *Imperfect Induction.* Sometimes, finally, the general truth is inferred from one or, at most, a few instances, the evidence consisting, not in the number of instances, but in the kind or quality of the instances. In this case one typical instance furnishes us with the knowledge of the cause of the event, and our conclusion is a statement of a causal relation. This we may style *Causal* or *Scientific Induction.* For example, one experiment with a coin and a feather in a vacuum tube will warrant us in concluding that in vacuo all bodies fall with equal velocity. We have by our experiment eliminated the interfering friction of the air, and therefore have produced a "typical" instance.

ENUMERATIVE INDUCTION AND SCIENTIFIC INDUCTION. Comparing Enumerative with Scientific Induction, we remark:

(1) Enumerative Induction, even when it is complete, may be of very little scientific value. I observe, for example, that the first book on

the shelf opposite me is a book on Dante, so is the second, the third, the fourth * * * and so on down to the last. Therefore, I conclude, all the books on the shelf are Dante books. This is a convenient summing up of my observations. There is no reason, however, why it should not be regarded as a true inference, because it is a progress from one form of knowledge to another, or from the "known under one aspect," to the "known under another aspect." Yet, it assigns no cause, and therefore is not of much use in science. It may, nevertheless, lead to the discovery of a cause. The conclusion "All those books are books on Dante," may lead to an inquiry concerning the owner and his tastes, and to the discovery that he is a lover of Dante.

(2) Enumerative Induction, when it is incomplete, gives only a probability, not a certitude. The probability is greater in proportion as the enumeration nears completeness. Thus, if I were to observe that the first apple in the basket is green, and the second, and the third, I should naturally expect the fourth to be green. But this is only a very slight probability. If, however, out of a thousand apples I discovered that nine hundred and ninety-nine were green, I should be almost certain that the last one would be green also. There is not, it should be remarked, in this process a mathematically accurate scale of probabilities. One new instance may upset a

whole series of observations or experiments. The goat is a ruminant, the cow is a ruminant, the sheep is a ruminant, and all these animals have horns. The camel, however, although a ruminant, has no horns. This last observation makes it utterly impossible for me to conclude that all ruminants have horns.

(3) Scientific or Causal Induction gives certitude. The reason why, having obtained a few properly tested instances, we infer a general statement is that in those few instances we have segregated, or abstracted, the *Cause,* and we know that the world in which we live is a world in which like causes are constantly producing like effects. We believe that the course of Nature is uniform, that if fire burns here and now, it will burn tomorrow, or at the other end of the world. If, then, by a series of experiments, or a few carefully conducted experiments, I discover that *A* is the cause of *a,* I am warranted immediately in saying that all *A* is the cause of all *a*. In other words, I am warranted in formulating a law to the effect that whenever *A* acts it causes *a,* and whenever *a* is produced it is produced by *A*. This, of course, implies that *A* is the sole cause of *a,* and supposes that nothing impedes its action. Once it was discovered by a long series of observations and experiments that the cause of malaria is a certain kind of mosquito, it became an estab-

lished principle in medicine and hygiene that all cases of malaria are due to the action of the mosquito and not to night air, miasma from swamps, etc.

Evidence on Which Inductive Conclusion Rests. It is very important to remark that, in the view of scientific induction which is here set forth, the conclusion, which is of its nature general, is drawn from a few instances, which are particular. The evidence, however, is not in the particular instances alone, but in the vast uniform order of Nature that lies behind the particular facts and is the work of an All-Wise Creator. The human mind brings to bear on the particular facts of experience the conviction that Nature is uniform, and so constructs an abstract, universal principle.

The Investigation of Facts. Scientific Induction presupposes the study of facts. It is only after we have investigated the facts that we can conclude inductively. Now, the investigation of facts takes place by *Observation* and *Experiment* and is sometimes aided by *Hypothesis* and *Classification*. These processes are subsidiary to Induction in the same way as Definition and Division are subsidiary to the Deductive processes.

Observation and Experiment. *Observation* and *Experiment* are the recognized means of ascertaining facts. They lead to the process of Induction. They agree in this that they are both concerned with the finding of instances. They differ, however, in one essential respect. In Observation we watch attentively the occurrence we are studying, but do not interfere with it. In Experiment, we *actively interfere* either to produce the phenomenon which we wish to study or to modify the circumstances or conditions in which the phenomenon occurs. The astronomer who trains his telescope on a distant star, the philologist who gathers up facts of interest to the linguist, the statistician and sociologist who examine and record facts relating to social life—all these are passive onlookers, and, therefore, are observers. The chemist who mixes several ingredients and notes the result, the physicist who subjects a metal to heat, electricity, etc., in order to discover and record the effects of these agencies, the physiologist who administers drugs and applies various stimuli to the nerve centers of animals for the purpose of finding out the effects on the various organs and functions—these are active investigators, experimenters.

Advantages of Experiment over Observation. The advantages of Experiment over Observation are well recognized:

I. By Experiment we can control the occurrence of the phenomenon we are studying: in Observation we must be content to wait until it occurs. The man who should trust to observation could not study the phenomena of electricity except when a thunder storm occurs: the experimenter can produce all the phenomena of a thunder storm a hundred times a day.

II. By Experiment we can isolate the phenomenon we are studying; this we cannot do by means of Observation. Thus, if we wish to ascertain what element in the atmosphere is necessary for animal life, we can by Experiment place an animal under the bell of an air-pump, exhaust the air, introduce separately oxygen and the other ingredients of the atmosphere and note the effect of each.

III. We can by Experiment vary the circumstances or conditions indefinitely. A chemist, for example, who has discovered a new substance can proceed to experiment on it and ascertain its various properties by combining it with other substances, subjecting it to the action of light, heat and other agents, etc. If he trusted to observation he should be obliged to wait until he found the new substance in those various combinations.

Advantages of Observation. On the other hand, Observation has its uses. There are sciences, such as psychology and astronomy, in which, although Experiment is employed, the investigator finds that the use of Observation is absolutely indispensable. There are some important phenomena of mental activity and there are many phenomena of the astronomical world which totally elude experiment. Besides, in other departments of inquiry, in medicine, hygiene, etc., we are often ignorant of the cause of the phenomena we are studying: we cannot, therefore, produce the phenomenon but must wait until it occurs, and observe it.

Rules of Observation and Experiment. There are rules for Observation and Experiment, which may be summed up as follows:

I. Observation and Experiment *should be precise.* It is often of the utmost importance to note the exact place, time, quantity, duration, etc., of the phenomenon. For this purpose we use instruments of precision, thermometers, micrometers, chronographs, the thermo-electric pile, etc. Our natural means of distinguishing heat from cold, for instance, would give us very vague and unsatisfactory results; whereas by means of a finely graded thermometer or thermo-electric pile we can distinguish very slight differences of temperature. In order to secure greater accuracy, it is advisable to take several observations, and, if there is any discrepancy, to take an average of observations.*

II. We should *attend only to material circumstances,* that is to say, to conditions which are likely to influence the effect materially. No

*Fowler, *Inductive Logic*, Oxford, 1892, p. 45.

physician nowadays, when noting the patient's symptoms, would think it necessary to observe the position of the planets, because the heavenly bodies are now recognized to have no material influence on bodily health. At the same time, too much caution cannot be exercised in regard to apparently irrelevant circumstances. Discoveries have often been delayed for whole centuries because some circumstance which really determined the effect was overlooked, or regarded as an immaterial or irrelevant circumstance.

III. The *circumstances* under which the Observation and Experiment are conducted *should be varied* as much as possible. A medical man when studying the effect of a certain drug will not confine his observations to one class of persons but will extend them to persons of all classes, conditions, constitutions, and habits of life.

IV. The *phenomenon* we are studying *should*, as far as possible, *be isolated* from other phenomena which may interfere with it. The action of gravitation is isolated, that is, removed from the interference of atmospheric friction, in the experiment in which a coin and a feather are made to fall through a certain space in a vacuum tube. There are, of course, circumstances which cannot entirely be eliminated, but, whenever they are such as to interfere with the phenomenon we are studying, they should be reduced to the minimum.

Rule V. We should be careful to *distinguish between what we actually observe and our inferences from the facts.* Most people confound these two very distinct sources of knowledge. The cross-examination of witnesses in court frequently brings out the fact that the witness fails to distinguish between what happened and his impressions of what happened. In scientific matters it is all-important to distinguish what we see and our inferences from what we see, and to keep these two apart.

Hypothesis. Besides Observation and Experiment, *Hypothesis* is to be counted among the operations subsidiary to induction. Indeed, no great progress can be made in the investigation of Nature without the use of hypothesis. A hypothesis is a supposition, or assumption, made on avowedly insufficient evidence in order to account for some fact or some law known to be real. Thus, it is a *Law* that bodies act as if they attracted one another directly as the mass and inversely as the square of the distance. The *Hypothesis,* by some scientists acknowledged to be without sufficient evidence, is that bodies attract each other, or, in other words, that each draws the others towards it. When a hypothesis has been proved to be the true explanation of a law or a group of facts, it is accepted as a *Theory*. Some, for example, regard what has just been called the hypothesis of gravitation as a theory of gravitation, and many who do not admit the theory of Evolution in the biological world accept the evolution hypothesis as a provisional assumption or, to use the common phrase, a "working hypothesis."

Conditions of a Valid Hypothesis. In order that a hypothesis be valid, or legitimate:

I. *It must not be at variance with the facts* already ascertained, or with inferences from them. If one were to suppose that the heavenly bodies move in perfect circles his hypothesis would be unscientific because it does not agree with the facts ascertained by actual observation.

II. *It must admit of proof or disproof.* At least, it must be of such a nature that it can be rendered more or less probable by further observation and experiment. To explain the law of gravitation by supposing that angels move the particles of matter towards one another is to introduce a hypothesis which can neither be proved nor disproved by scientific research.

III. It must, if it is an explanation, *apply to all the facts* to be explained, and if it assign a cause, it *must assign a cause adequate to produce all* the phenomena. Thus, according to some scientists there are two rival theories of the maintenance of solar heat, the theory of combustion and the theory of meteoric impact.

Of these, they say, the latter is to be preferred, because, while the former assigns a cause, it does not assign an adequate cause.

Uses of Hypothesis. The uses of a hypothesis are various:

I. It may, by subsequent proof, be *established as the true theory or law.* Thus, many who first adopted Evolution as a hypothesis to explain the facts observed in the realms of animal and plant life maintain that there is now at hand sufficient evidence to establish Evolution as the true theory of the origin of the present flora and fauna. The undulatory theory of light is now accepted as proved; before the proof was considered sufficient, what is now a theory was a mere hypothesis.

II. A hypothesis may be of service in *pointing the way to the true theory.* Kepler, for ininstance, having observed the facts regarding the courses of the planets, formulated twenty-nine different hypotheses, each of which he successively abandoned until he finally proved that the theory of planetary motion is the hypothesis that each planet moves around the sun in an elliptical orbit with the sun at one of the foci.

III. Every hypothesis whether it prove eventually to be true or false, *links together the facts already observed.* Like a string running through a number of beads, it holds the facts in a definite order and thus preserves them in convenient form. This is the part which the Ptolemaic hypothesis played. It served by its assumption of cycles and epicycles to preserve the results of all the observations made regarding the stars down to the time when it was supplanted by the Copernican theory.

Classification. Among the processes subsidiary to Induction is also to be enumerated *Scientific Classification.* Classification is to Induction what Division is to Deduction. In a sense, however, it is the inverse of logical Division. For, while Division starts with a large group, or class, and divides it into smaller groups, or classes, Classification starts with smaller groups, or with individuals, and arranges them in larger groups and classes. The requirements of scientific Classification are summed up in one principle: a scientific classification should first gather individuals into natural groups and then arrange these groups by proper coordination and subordination into a natural series. A group is natural, or a series is arranged naturally, when the most important differences and similarities are selected as the basis of grouping and arrangement. Thus, in classifying animals, the most important difference, structure, is made the basis of a natural classification.

When the classification is purely *artificial,* that is to say, is intended, not for the purpose of discovering laws of nature, but for some immediate practical purpose, the principle of arrangement may be entirely extrinsic and unimportant. The words in the English language may be classified naturally according as they are Anglo-Saxon, Celtic, Norman-French, or Latin. When they are classified artificially, for the purpose of ready reference, they are grouped alphabetically according to their initial letters.

CHAPTER XVIII

Methods of Scientific Induction

SCIENTIFIC INDUCTION. The investigation of facts by means of observation and experiment, the orderly arrangement of them by means of scientific classification and the provisional explanation of them by means of hypotheses, lead up to the problem of *Scientific Induction,* the discovery of the cause. The establishment of the cause is a process of synthesis, comparison, abstraction, or generalization, that is, a putting together of individual facts, a comparison of them, for the purpose of eliminating the irrelevant and accidental, and the affirmation of a causal relation which, considering the uniformity of nature, must be general. Let us suppose that the problem is to find the cause of nocturnal dew. First we observe a number of objects, wood, metal, mineral, etc., exposed to the atmosphere during the night. These facts are gathered together. Next, they are compared and sifted. Various hypotheses may be tried to explain why the dew is formed more copiously on some objects than on others: the color, for instance, may be supposed to have some effect. Thirdly, we single out the phenomenon of radiation of heat and, adopting the hypothesis that dew is precipitated on surfaces which radiate heat more rapidly, we test this hypothesis by the facts observed. We have thus abstracted from the other conditions this one condition, and, finding that it is the cause of the phenomenon we are studying, we are warranted by the uniformity of nature, in generalizing the causal relation and saying that the cause of dew is the lower temperature of the object compared with the surrounding atmosphere. Indeed, we may make our conclusion still wider, and affirm that not only nocturnal dew but also similar phenomena, such as the deposit of moisture on a cold glass suddenly brought into a warm room, are due to the same cause. We have, then, in scientific Induction a general conclusion drawn from the evidence of particular facts. The facts, however, could not prove the general conclusion if it were not for the abstractive power of the mind and the background of uniformity of nature into which the particular facts are fitted.

Uniformity of Nature. There is much discussion among logicians regarding our conviction that the course of nature is uniform. or, in other words, that *like causes produce like effects.* Some hold this conviction to be an instinct; some hold it to be itself an induction; some hold it to be the result of experience.

There is no doubt that experience furnishes us the data, by teaching us what is a cause and what is an effect. But, once we understand what is meant by these terms, we are able to judge immediately and analytically that "Like causes produce like effects." The judgment is, therefore, an analytical judgment like the judgment that "A straight line is the shortest distance between two points."*

SCIENTIFIC INDUCTION IS A TRUE INFERENCE. Scientific Induction is, consequently, a process of Reasoning. From premises which are facts, it derives a conclusion which is a general statement concerning a cause. It is a true inference, because it brings out explicitly an item of knowledge which is implicitly contained in the premises. And it does not contravene the general law of inference which says that *the conclusion may not be wider than the premises.* Because the justification of the conclusion is not in the particular facts, as particular, but in the universal, uniform, and consistent order of nature. As soon as we bring out of the observed facts the relation of causation, we are dealing with the universal and necessary, not with the particular and contingent, we are dealing not with this precipitation of moisture on this particular piece of metal but with precipitation of moisture on a substance which radiates heat rapidly.

*Cf. Chapter VI, p. 82.

Does it Differ from the Syllogism? If, now, we are asked whether the process of Scientific Induction differs from the syllogism, we may answer that it does, in so far as it starts with the particular facts, while the syllogism starts with a general principle. Moreover, it is justified by an appeal to the Uniformity of Nature, while the syllogism is justified by an appeal to the *Dictum de Omni.* Whether this constitutes an essential difference will depend on what we mean by an essential difference between two processes both of which are true inferences, though each employs its own peculiar method. A scientific Induction may, by a good deal of circumlocution, be cast into syllogistic form, but it does not fall into that form naturally. Thus, the discovery of the cause of dew may be expressed thus:

All causal relations in nature are constant;
Between rapid cooling of the surface and the deposit of dew there is a causal relation;
Therefore, this causal relation is constant.

The proof of the minor would be an eliminative syllogism

The cause of the deposit of dew is either color, weight, texture, or rapid cooling of surface;
It is not color, weight, etc.
Therefore, it is the rapid cooling of the surface

But, when all the evidence is thus reduced to syllogistic form, there remains the conviction that somehow the peculiar strength of the argument has not been expressed at all. In a word, Scientific Induction does not need to be cast into syllogistic form in order that its validity be apparent.

METHODS OF SCIENTIFIC INDUCTION. The *Methods* of Scientific Induction are generally given in the form in which they were enunciated and arranged by John Stuart Mill (1806-1873).* In each case Mill gives (a) The canon, (b) The formula, (c) The principle, and (d) Examples.

(A) THE METHOD OF AGREEMENT. (a) *Canon:* "If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon."†

(b) *The Formula* of this method is $A\ B\ C$ followed by $a\ b\ c$; $A\ D\ E$ followed by $a\ d\ e$; therefore, A is the cause of a. $A\ B\ C\ D$ and E are antecedents, $a\ b\ c\ d$ and e are consequents.‡

(c) *The Principle* is that whatever can be

*See Turner, *History of Philosophy*, pp. 615 ff.

†*System of Logic*, III, 8, 1.

‡It is necessary to warn the student against Mill's false theory of causation. Mill defines a cause as the *invariable antecedent* of an event. He rejects the scholastic view, according to which the cause *influences* the effect and the effect *depends* on the cause.

eliminated from the phenomenon cannot be part of the cause. *A B C* is the first instance, that is, a group of antecedents, followed by *a b c*. The second instance, *A D E* followed by *a d e*, eliminates *B* and *C* without prejudice to *a*. Therefore, neither *B* nor *C* can cause *a*. And the first instance just as effectually eliminates *D* and *E*. Therefore, *A* is the antecedent which causes *a*.

(d) *Example*, Jevons in his "Principles of Science"* gives an example of the use of this method when he tells how Sir D. Brewster "accidentally took an impression from a piece of mother-of-pearl in a cement of resin and bees'-wax, and finding the colors repeated upon the surface of the wax, he proceeded to take other impressions in balsam, fusible metal, lead, gum arabic, isinglass, etc., and always found the iridescent colors the same. He thus proved that the chemical nature of the substance is a matter of indifference, and that the form of the surface is the real condition of such colours."

The "*Practical Imperfection*" of this method is due to the fact that a phenomenon may have several causes. In the formula given above, *B* may be the cause of *a* in the first instance and *E* in the second. This imperfection may, however, be remedied by the multiplication of instances. It is possible that *a* may have two causes; it is

*London, 1892, p. 421.

not probable that it has ten, and it is almost impossible that it should have a hundred. The possibility of a plurality of causes is eliminated by increasing the number of instances observed.

(B) Method of Difference. (a) *Canon:* "If an instance in which the phenomenon under investigation occurs and an instance in which it does not occur have every circumstance in common save one, that one occurring only in the former: the circumstance in which alone the two instances differ is the effect or the cause or an indispensable part of the cause of the phenomenon."*

(b) *The Formula* for this method is: *A B C* followed by *a b c*, *B C* followed by *b c*; therefore *A* is the cause of *a*. Here, again, *A B C* are the antecedents and *a b c* the consequents in the first instance observed, *B C* the antecedents and *b c* the consequents in the second instance observed. *A*, the antecedent absent in the second instance, is the cause of *a*, the consequent absent in the second instance.

(c) *Principle.* The principle is that whatever cannot be eliminated without prejudice to the phenomenon must be connected with it in causation.

(d) *Examples:* "It is scarcely necessary," writes Mill, "to give examples of a logical process to which we owe all the inductive conclu-

*System of Logic, III, 8, 2.

sions we draw in early life. When a man is shot through the heart, it is by this method that we know that it is a gunshot which killed him: for he was in the fullness of life immediately before, all the circumstances being the same, except the wound.''*

When a piece of paper is thrown into the fire, we ascribe its sudden combustion to the action of the fire, because the sudden increase in temperature is the only new condition to which it is exposed, and we feel sure that any change which takes place is due to that cause.

When a gong is made to vibrate in an inverted glass bell we hear the sound. If the glass bell is now placed under an airpump, and, the air having been exhausted, we fail to hear the sound, we conclude that the air had a determining influence on the transmission of sound waves.

The Method of Agreement and the Method of Difference may be combined in one series of experiments, thus giving us the *Joint Method of Agreement and Difference.* For instance, in a particular part of the country we find that when it rains the wind usually blows from the east. This immediately gives rise to the suspicion that the east is for that locality the "rainy quarter." If, going farther in our investigation, we find that the

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**System of Logic*, ibid.

west, south, or north wind never accompanies rain, we have established a causal connection between the rain and the east wind.

(C) METHOD OF RESIDUES. (a) *Canon:* "Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents."*

(b) *Formula:* If it be known by previous inductions that *A B C* is invariably followed by *a b c*, and if we further discover that *B* is the cause of *b*, and *C* the cause of *c*, it follows that *A* is the cause of *a*.

(c) *Principle:* Given that the total result is due to a certain number of antecedents, and a certain part of the result is due to a certain part of the antecedents, the residue of the result must be due to the remainder of the antecedents. In reality, this is a deductive, not an inductive process. Yet, it is by this method that some of the most important scientific discoveries are made.

(d) *Examples:* Mill, quoting from Herschel's "Outline of Astronomy," writes, "Almost all the greatest discoveries in Astronomy have resulted from the consideration of residual phenomena of a quantitative or numerical kind. . . . It was thus that the grand dis-

**System of Logic,* III, 8, 5.

covery of the precession of the equinoxes resulted as a residual phenomenon from the imperfect explanation of the return of the seasons by the return of the sun to the same apparent place among the fixed stars.''*

In chemistry, also, new elements, such as argon, have been discovered by the examination of residual phenomena.

Fowler† quotes the following example: "On comparing the accounts of live cattle and sheep annually sold in Smithfield market for some years past, it appears that there is a large increase in cattle, while the sheep are nearly stationary. The consumption of meat in London may be presumed to have increased, at least in proportion to the increase of its population; and there is no reason for supposing that the consumption of beef has increased faster than that of mutton. There is, therefore, a residuary phenomenon, viz., the stationary numbers of the sheep sold in Smithfield, for which we have to find a cause. This cause is the increased transport of dead meat to the metropolis, owing to steam navigation and railways, and the greater convenience of sending mutton than beef in a slaughtered state."

(D) METHOD OF CONCOMITANT VARIATIONS. (a) *Canon:* "Whatever phenomenon varies in

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**System of Logic*, III, 9, 5.
†*Inductive Logic*, p. 180.

any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation."*

(b) *Formula:* Mill does not give a formula for this Method. We may, however, adopt the formula suggested by some logicians. If $A1$, $A2$, $A3$ are variations of the antecedent phenomenon and $a1$, $a2$, $a3$ are variations of the consequent, and if we find that $A1\ B\ C$ is followed by $a1\ b\ c$, $A2\ B\ C$ by $a2\ b\ c$, $A3\ B\ C$ by $a3\ b\ c$, then A is the cause of a.

(c) *Principle:* This Method is in principle akin to the Method of Difference. When a phenomenon can not be made to disappear altogether, but occurs in different degrees of intensity or in different quantities, the corresponding variations in another phenomenon are an indication that there is a causal connection.

(d) *Examples:* The Arabians, as far back as the ninth century, knew that the moon influences the tides, and the medieval physicists proved the causal influence by showing that the variations in the tides correspond to the phases of the moon.

It was by observing the variations in the height of the column of mercury in a barometer

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***System of Logic*, III, 8, 6.

that Pascal established the causal connection between the height of the column and the weight of the atmosphere.

Physiology affords many examples of the successful use of this method. For instance, the effect of the blood circulation in the brain on mental life in men and animals is established by noting how variations in the blood flow correspond to variations in the mental functions. This method is also applied with great success to the study of social phenomena and institutions. Whenever any institution, custom, or condition of society is found to vary concomitantly with the variations in some other institution, custom, or condition, there is immediately a strong probability that we are dealing with a case of causal dependence. For instance, some statisticians maintain that the criminal records show an increase of murder in countries where capital punishment has been abandoned, and no corresponding increase in countries where it is still maintained. If these facts are as represented, one is forced to conclude that capital punishment tends to diminish the number of murders. Again, Protestant historians sometimes maintain that wherever the Catholic Church has held sway illiteracy prevails, and that where the influence of the Church is diminished education flourishes. Before concluding, however, that, therefore, the Church has a deleterious influence on education they should show that there is no other factor, such as climate, racial defects, etc., which interferes with the action of the Church. They should show, too, that in the countries in question the Church is free to follow its public policy. If, however, no such interfering causes are demonstrated, and if the facts are as represented by Protestant historians, the Church would be proved to be the enemy of education. If, on the contrary, the Catholic historians can show that the facts are quite different, the opposite conclusion is warranted.

CRITICISM OF THESE METHODS. The high claims which Mill in his *System of Logic* makes for his Methods of Inductive Inquiry are not admitted by the majority even of those who follow him in their treatises on logic. The most serious criticism which they offer is that the methods take for granted the very thing which is most difficult to discover, namely the reduction of phenomena to formulae such as *A B C, a b c,* which are here presented to us. Indeed, Nature is far more complex than any set of formulae, and the greatest of all difficulties in the investigation of Nature is the reduction of phenomena to simple terms and the segregation of antecedents from consequents. Nevertheless, the Methods serve as models according to which our observations and experiments may be organized and presented in the most concise form. They are not themselves forms of Inductive Reasoning, but they show how the evidence for our Inductive Reasoning may be formulated so as to lead to the discovery of a cause.

The objection that no great discovery can be traced to the use of these methods is less serious. It is true, as the opponents of the Methods contend, that many discoveries are made, as we say, "by chance." Roentgen's discovery of the Rays with which his name is associated was entirely "accidental." There is, however, much truth in Mill's reply that, while discoveries are often due to fortuitous circumstances, usually to something which "happens" while the investigator is looking for something else, it is only the man who is trained in the use of scientific methods that can see the value of the "accident," take advantage of it, repeat the experiments so as to observe the antecedents of the accidental occurrence and so, eventually, discover the cause of it. This consideration justifies the retention of the Methods in Logic as a means of training the mind in scientific inquiry.

From the point of view of philosophical criticism the most important consideration is this: Mill wrongly contends that in Induction we argue from particulars to a new particular instance. An examination of his own Methods shows clearly that not one of them is valid unless we appeal to general principles, such as the Uniformity of Nature. It shows also that unless we admit, as Mill does not, the power of the human mind to abstract general principles from individual facts, we cannot by the aid of these Methods establish a general truth, such as that *A* is everywhere and always the cause of *a*.

CHAPTER XIX

Fallacies

FALLACY, SOPHISM, AND PARALOGISM. The word *Fallacy,* as it is commonly used, means any false opinion, inaccurate statement, confusion of ideas, or even clumsiness of expression. More strictly, a fallacy is any violation of a logical principle disguised under a show of validity. More strictly still, a fallacy is an argument which, while apparently valid, really violates some logical principle. The chief synonyms for fallacy are *Sophism* and *Paralogism.*

CLASSIFICATION OF FALLACIES. It is, perhaps, hopeless to expect a complete and scientific classification of the ways in which men may arrive at error. Those ways are infinite in number and variety. Still, if we confine our attention to the principal sources of error in reasoning we may arrange the different fallacies as follows:

I. *Fallacies incident to Deduction.*

(A) *Fallacies arising from language*
(B) *Fallacies arising from some other source*

(a) *Purely logical fallacies*
(b) *Semi-logical fallacies*
(c) *Material fallacies*

II. *Fallacies incident to Induction.*

I. Fallacies Incident to Deduction. The fallacies which occur in deductive reasoning either arise from the language used or from some other source, namely, either from the non-observance of the laws of correct reasoning, or from a misapprehension of the Middle Term, or from the matter, that is, the content of the argument. Fallacies arising from the language are called *Fallacies of Diction.* All others are *Extradictional Fallacies.* These last are *Purely Logical* when they arise from the violation of the rules of the syllogism, *Semi-Logical* when they arise from a misunderstanding of the Middle Term, and *Material* when they arise from the matter, or content, of the argument.

A. Fallacies of Diction, as enumerated by Aristotle, are six.

1. *Equivocation* is a fallacy which arises from the multiple meanings of one and the same term. For instance:

All cold can be expelled by heat;
John's illness is a cold;
Therefore, it can be expelled by heat.

Sometimes, as in this example, the fallacy is nothing more than a contemptible pun. Sometimes, however, especially in arguments concerning historical and sociological subjects, a term, such as "prosperity," "good government," "education," may be taken by one person in a strictly material or temporal sense, while it is taken by another person in a sense wide enough to include intellectual and spiritual interests. The result in such cases is ambiguity and fallacious argumentation.

2. *Amphibology* means ambiguity of structure. In this case, the misunderstanding arises, not from any one word but from the structure of the sentence. A well-known example is Shakespeare's sentence "The Duke yet lives that Henry shall depose," which could mean either that the Duke shall depose Henry or that Henry shall depose the Duke. Minto* gives the following ludicrous example:

What he was beaten with was what I saw him beaten with;
But I saw him beaten with my eye;
Therefore, he was beaten with my eye.

In English, the position of adverbial clauses is often a source of unconscious humor, as in the example, so often quoted, of the newspaper headliner who wrote "A farmer blows his brains out after taking an affectionate farewell of his family with a shotgun."

***Logic, Inductive and Deductive*, New York, 1904, p. 227.

3. *Composition* is the fallacy which arises when we take collectively what is true only distributively. For instance,

All the angles of this triangle are less than two right angles;
A, B, and C are the angles of this triangle;
Therefore, $A+B+C$ are less than two right angles.

4. *Division* is the inverse of the preceding fallacy. It occurs whenever we assert a thing to be true distributively because it is true collectively. For example

The twelve men of the jury are not likely to err;
John Smith is one of the twelve;
Therefore, John Smith is not likely to err.

5. *Accent.* This Fallacy occurs when the transfer of the accent from one syllable to another gives the word a different meaning. Nestorius, who denied that Christ is God, denied also that Mary is the Mother of God. At a certain synod he is said to have subscribed to a formula declaring that she is θεοτόκος, which, with the accent on the penultimate, means "Mother of God." Later on, however, wishing to retract this statement, he declared that he meant the accent to be on the antepenultimate, and that he meant merely that she is θεότοκος, "the daughter of God." Latin authors give the

following example: "Qui occidit est causa mortis; atqui sol vespere occidit. Ergo, sol est causa mortis." In English, the only transfer of accent that can give rise to ambiguity is the transfer of the emphasis or stress from one word in a sentence to another. The reader who lays undue stress on the last word of the sentence "Thou shalt not bear false witness against thy *neighbor*" implies that the commandment does not condemn false witness against strangers.

6. *Figure of Speech.* Any transition from the literal to the figurative use of words, or *vice versa,* is fallacious. But, by the fallacy of Figure of Speech Aristotle means any fallacy which arises from the supposition that words similar in form are similar in meaning. "Whatever is rough can be made smooth by the use of a file; his manners are rough; therefore, they can be made smooth by the use of a file"—is an example of transition from the literal to the figurative. To argue that because what is "imaginary" is unreal, an "image" made of wood or stone must be unreal, is to suppose that words similar in form are similar in meaning.

B. Extradictional Fallacies. These, as we pointed out above, are either (a) purely logical, (b) semi-logical, or (c) material.

(a) *Purely Logical Fallacies* arise from the violation of the rules of right reasoning. Non-distributed Middle, Illicit Process of the Major, and Illicit Process of the Minor are the principal kinds of purely logical fallacies.*

(b) *Semi-Logical Fallacies* arise from a misunderstanding of one of the terms, usually the Middle Term. It is different from the purely dictional fallacy of Equivocation: it is not quite dependent on a verbal confusion, and yet it is partly dependent on words. There are two kinds of semi-logical fallacies:

1. *Fallacy of Accident.* This arises whenever we confuse together the essential and the accidental qualities or characteristics of an object. The following is a well-known example: "What you bought yesterday you ate today: you bought raw meat yesterday. Therefore, you ate raw meat today." The first premise refers to the substance, the second to an accidental state of that substance. Similarly, all argument from the abuse of a thing to the condemnation of the use of that thing is a fallacy of accident. The Romans had a very sound principle of law: "*Abusus rei rei usum non tollit.*" It is fallacious to argue "He who knowingly and willingly drinks an intoxicant is guilty of sin. But, wine is an intoxicant. Therefore, he who knowingly and willingly drinks wine is guilty of sin." Wine is an intoxicant only in excess. To

*See Chapter XIII.

abstain from wine altogether is either a matter of worthy devotion to principle, a matter of good example, of personal asceticism, or in exceptional cases, a matter of avoiding the occasion of sin. It is not a question of avoiding sin.

2. *Confusion of Absolute and Qualified Statement.* This resembles the fallacy of Accident. It consists in failing to discriminate between an unqualified assertion and an assertion qualified by some restriction. For example "It is unlawful to take another man's life. But the soldier in battle takes another man's life. Therefore, what the soldier does in battle is unlawful." Here, the absolute statement that it is unlawful to take another man's life must be qualified before it is true. For, all moralists hold that to take another's life in lawful warfare, or in self-defence, or in the name of the law (in legal execution) is not wrong. The social reformer who clamors for universal education as the cure of all the ills of the social body, and condemns ignorance as the only evil, often forgets that the proposition "Education is a benefit to mankind" needs to be qualified in order to be true. It must be the right, not the wrong, kind of education, not merely the ability to read the newspapers and cheap magazines, but education which is spiritual and moral and develops the mind so as to fit it for the enjoyment of the best kind of literature.

(c) *Material Fallacies.* These are generally said to arise from the matter, or content, of the argument. That is to say, they do not arise from the language nor from the process of logic employed, but are seen to be fallacious as soon as the meaning and intent of the argument are studied.

1. *Petitio Principii,* or "Begging the Question." This is a fallacy which occurs whenever we take for granted something which we should not take for granted, that is, either the conclusion to be proved or a part of it. Sometimes, indeed, the sophist brazenly assumes the whole conclusion, though he may disguise it by casting it into different phraseology: when, for example, in the attempt to prove that a certain measure is "unconstitutional," he starts by affirming solemnly that "Whatever is contrary to the principles of the constitution is unconstitutional" and that "this measure is contrary to the principles of the constitution." The assertion made in the Minor is precisely what the speaker has undertaken to prove. "The volume of a body diminishes when it is cooled, because the molecules then become closer" and "opium produces sleep, because it has a soporific effect," are examples of the same kind of fallacy. Sometimes a single word introduced without sufficient warrant in the statement of the proposition to be proved begs the whole ques-

tion. Thus the legislator who begins by referring to the measure under discussion as "this unconstitutional measure," by the use of the word "unconstitutional," which he fails to justify, takes for granted the very thing he is expected to prove. Words of this kind are called "question-begging epithets." Sometimes the *petitio principii* takes the form of a "reasoning in a circle." An example of this is cited from Plato, who, in the "Phaedo," demonstrates the immortality of the soul from its simplicity and in the "Republic" proves the simplicity of the soul from its immortality.

2. *Ignoratio Elenchi*, or "Irrelevant Conclusion," consists in evading the point at issue and, instead of proving the conclusion that should be proved, trying to establish some other conclusion. Thus, some opponents of classical education triumphantly ask "To what practical use can a boy put a knowledge of Latin and Greek?" thereby substituting for the real issue, namely the value of Education, an irrelevant issue, the practical value of Education.* A very common form of Irrelevant Conclusion is the *Argumentum ad Hominem*. This consists in turning aside from the discussion of the question at issue and directing attention to the personality of one's opponent, accusing him of inconsistency, or attacking, or ridiculing his per-

*Cf. Spencer, *Education*, Chapter I.

sonal character or appearance. "No case; abuse the plaintiff's attorney," is the well-known advice given to a juvenile lawyer by an older practitioner, more learned in the art of wheedling a jury than in the science of law. To attribute base motives to one's opponent, to reflect on his manner of living, to attack his political or religious convictions when these are not the subject of debate, may have the effect of discouraging him or bringing him to confusion, but do not affect the merits of the cause he is advocating. Similar to this are the *Argumentum ad Populum,* in which the speaker or writer appeals to the prejudices and passions of his audience or his readers; the *Argumentum ad Ignorantiam,* in which the sophist counts on the ignorance of his audience and takes advantage of it to conceal the weakness of his own case or misrepresent the case of his opponent; the *Argumentum ad Verecundiam,* in which he attempts to put his opponent to shame by urging the superior weight and dignity of authority on his own side: "Who are you," he cries, "to set yourself up as an authority against such men as Huxley, Spencer, and Darwin?" Finally, there is the *Argumentum ad Baculum,* the appeal to physical force, of which it has been said "To knock a man down when he differs from you in opinion may prove your strength, but hardly your logic." Shouting one's phrases in stentorian tones, thumping the pulpit, shaking one's first at an imaginary opponent, and other forms of violence in manner are akin to the fallacy of the *Argumentum ad Baculum.*

3. *False Cause,* or *"Non causa pro causa,"* is sometimes defined as the assigning effects either to an imaginary cause or to a real thing which is not a cause of the effect in question. *"Post Hoc, Ergo Propter Hoc,"* or assuming that an event which succeeds another is, therefore, caused by it, is a common form of fallacy. When, for instance, the "Republic," sailing from New York on Friday, was rammed in a fog off Nantucket on Saturday, many persons imagined that it was because she sailed on Friday.

More correctly, however, the *"Non causa pro causa"* does not refer at all to "cause" in the scientific sense, but in the Latin juridical sense of "case" or "contention." It consists in attempting to show that certain absurd conclusions follow from our adversary's case, or contention, whereas they follow from entirely different premises. Let us suppose that the speaker is arguing in favor of capital punishment for murder. The sophist answers "Your contention is absurd, because, if the death penalty should be enforced as a deterrent in the case of murder it is an equally efficacious deterrent in the case of theft, and therefore, in your contention a man should be hanged for picking a pocket''—whereas the speaker defended the death penalty on entirely different grounds, for example, on the ground that the punishment should be proportioned to the crime.*

4. *Complexity of Question,* or ''Many Questions.'' This fallacy was a common expedient of the Greek sophists in their effort to put an opponent to confusion.† It consists in putting a question apparently single but really multiple, and insisting on an unqualified ''yes'' or ''no'' for an answer. ''Have you given up the habit of telling lies?'' is an example. If you answer ''yes,'' you imply that you have had that habit, if you answer ''no,'' you intimate that you still tell lies. Of course, there is really no fallacious argument here. There is merely a trick of the rhetorician which one can evade by insisting on one's right to resolve the question into its parts and answer each separately.

II. Fallacies Incident to Induction. The mistakes which may occur in arguing from particulars to a universal, or in discovering the cause of a phenomenon may be classified as follows:

(A) *Fallacies of Observation:*
   (a) *Fallacies of Non-Observation,*
   (b) *Fallacies of Mal-Observation,*

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*Cf. Joyce, *Principles of Logic,* p. 281.
†Cf. Turner, *History of Philosophy,* p. 71.

(B) *Fallacies of Generalization:*
  (a) *Illicit Generalization,*
  (b) *False Analogy.*

(A) *Fallacies of Observation* occur in the investigation of facts which supply the evidence for the inductive process.

(a) *Non-Observation.* This consists in neglecting or overlooking instances which should be observed or circumstances which have a determining influence. The commonest form of this fallacy is the dwelling on positive instances exclusively, and overlooking negative, or contrary, instances. This is how many prevalent superstitions are sustained. If, for example, one dreams of an event as about to happen, and it does happen, the occurrence makes a deep impression on the mind, and is remembered. But, if the event dreamed of does not happen, no mental record is made of this negative, or contrary, instance. Similarly, in judging the characters of our neighbors, the person whose sympathies are strong sees only the good qualities of his fellowman and the person of strong antipathies sees only the faults. Both overlook a number of things which should be counted in a perfectly dispassionate estimate. It was by neglecting to observe contrary instances that Aristotle persuaded himself of the truth of the common opinion that on the seashore no animal ever dies except during the ebbing of the tide.

Sometimes, the non-observation occurs regarding circumstances which are important. In former ages it was customary to cure a wound by applying a "sympathetic powder" to the weapon that caused the wound, the wound itself having been first "brought together, carefully bound up with clean linen, and above all, let alone for seven days." Of course, it was this circumstance and not the "sympathetic powder" that effected the cure.

(b) *Mal-Observation.* By this is meant the wrong interpretation of sense impressions. A very usual form of mal-observation arises from the confusion of our own inferences with the impression actually received. For many centuries people believed that they "saw" the sun come up from behind the horizon and go down again; whereas this was really an inference from what they observed. All the illusions of the senses, if not detected, are instances of mal-observation.

(B) *Fallacies of Generalization.* This consists in overrating the evidence of the facts observed, and drawing a conclusion which the facts do not warrant.

(a) *Illicit Generalization* consists in drawing a general conclusion from an insufficient number of facts or in generalizing from an instance as typical, when the instance is not typical at all. As an example of the latter we may cite Aristotle's famous saying that the cranium of a dog has only one bone. Probably the sample which he observed was a cranium in which the sutures had become obliterated by age. As an example of the former we may cite the false generalization that "All ruminants have horns," which may find justification in the observation of a limited number of instances, cows, goats, sheep, deer, etc., but is not universally true.

(b) *False Analogy.* An analogy, as we shall see, in the next chapter, is based on the conviction that if *A* resembles *B* in possessing the attributes *a, b, c, d,* and if *B* is found to possess the attribute *e,* it follows that *e* is also an attribute of *A*. Analogy is at best a weak form of argument, and when it is pushed too far it is always a source of error. The body politic for instance, is compared with the individual body. The analogy holds to a certain extent; when urged beyond the proper limits it becomes a false analogy and leads to error.

CHAPTER XX

Applications of Logic

Principal Applications of Logic. It is obviously impossible to give here, even in outline, all the various applications of logic to the different departments of knowledge. To enumerate what may be considered the principal applications of this science, and to show in what way logic is turned to account in the several branches of study, will be the subject of this chapter.

Definition and Division. In every branch of study, and indeed, in every department of thought, the rules of logic regarding Definition and Division are applied. Clearness in our mental images, order among our ideas, and the orderly arrangement of any study that is undertaken: these are secured by observing the rules of logic concerning Definition and Division.

Opposition and Conversion. So, too, in respect to the rules of Opposition and Conversion of propositions. Consistency among our own convictions, and the ability to detect inconsistency or inconsequence among the various statements which are made about some subject, are attained by the observance of the rules laid down in Chapters IX and X.

Deductive and Inductive Sciences. When, however, we come to the more particular applications of logic, we find that the sciences are usually divided into two great classes, the Deductive and the Inductive. The Deductive are the so-called Abstract Sciences, such as Geometry, Ethics, Metaphysics, which deal with principles; the Inductive Sciences are the so-called Concrete Sciences, such as Natural Science, Sociology, Philology, History, which deal with facts. This differentiation of the sciences is, in the main, correct. We should not, however, draw a hard and fast line between Deductive and Inductive sciences. Because, we shall see, we find that in most of the sciences Deduction and Induction are used alternately. Retaining, therefore, the designation "Deductive" and "Inductive," we shall take them to mean the sciences which are predominantly Deductive and Inductive, respectively.

Concrete and Abstract Knowledge. Knowledge becomes scientific as soon as it becomes universal; that is to say, knowledge of individual and concrete facts is not scientific until it is related by the mind to other facts or to general principles. In the physical sciences, it is true, we begin by studying individual facts or concrete instances; but the aim is to proceed from them to a knowledge of Nature's laws. In Mathematics, on the contrary, we begin with principles, definitions, or axioms, which may, indeed, be illustrated by reference to concrete examples, but have a meaning and an application independently of the concrete instances. "A circle," "a triangle," "a square" are defined in general terms, although they may be illustrated by reference to "this circle," "this triangle," "this square." "Rousseau, in his *Émile,* tells us that we should teach a child geometry by causing him to measure and compare figures by superposition. While a child was yet incapable of general reasoning, this would doubtless be an instructive exercise; but it never could teach geometry, nor prove the truth of any one proposition."* The procedure in mathematics is, therefore, from general principles to less general conclusions, namely, by Deduction. Similarly, in metaphysics, which treats of the nature and general properties of reality. Our notion of reality is derived, no doubt, from experience, from the consideration of individual, real objects. But at the very beginning of the science we frame general principles from which, by Deduction, we infer the truths constituting the body of doctrine of metaphysics. Again, in ethics, our notions of "right" and "wrong" are, for the most part, rendered intelligible to us by our natural, or, as some say, "instinctive" judgment concerning the morality of such and

*Jevons, *Principles of Science*, Lond., 1892, p. 233.

such an action presented to us in its concrete setting. No sooner, however, have we formed these notions of right and wrong than we proceed to frame general principles, such as "Good is to be sought after; evil is to be avoided," and our progress in the study of ethics means the continued application of these principles to concrete cases—a deductive process. To take a tangible example from ethics, our idea of "justice" is built up from the study of concrete instances of "just actions" and a comparison of them with "charitable" acts, "kind" acts, "courteous" acts, etc. As soon as our idea of justice is built up, we are in a position to define justice as "A virtue which constantly inclines one to give every fellowman what is strictly due him." Having acquired this principle, we proceed to apply it, deductively, to a particular instance, and to argue, for example, "The ten dollars which I owe the baker is strictly due him; therefore, the payment of ten dollars is a matter of justice." Or "My neighbor has a strict right to his good name and reputation; therefore, to deprive him of it would be unjust."

Induction in the So-Called Abstract Sciences. In these sciences, therefore, which are generally called Deductive, the first steps are made, sometimes, by an inductive process. Sometimes, no induction is necessary, that is, when the primary principles of the science are self-evident. But whether the primary principles are inductively established or are assumed as self-evident, progress from the first principles to the conclusions of the science is deductive in the case of mathematics, metaphysics, and ethics.

Inductive Sciences use Deduction. The sciences which are called Inductive are so called because in them the chief business of the inquirer is from facts or concrete instances to build up laws or principles. In physics, in chemistry, in biology, in the study of languages, in the sociological group, in history, we begin with facts, and by induction arrive at a knowledge of the laws which explain the facts. Nevertheless, the inductive sciences do not entirely dispense with deduction. As soon as the laws explanatory of the facts are established, they are compared with other laws, tested by hypothesis, and supplemented by analogy, and finally pass into the region of mathematical reasoning, where the process becomes purely deductive. Jevons writes "It will now be apparent, I think, that though observation and induction must each be the ground of all certain knowledge of Nature, their unaided employment could never have led to the results of modern science."*

Both Induction and Deduction are used. A purely inductive science is, therefore, as little to

Lessons in Logic, London, 1894, p. 263.

be dreamt of as a purely deductive science. All departments of study use both processes. The study of Law, for example, furnishes an additional proof of this assertion. Law, as an enactment of regularly constituted authority, divine, ecclesiastical, or civil, is generally expressed in universal terms. The process from a universal legal principle to a particular application of it is deductive. The law, for instance, provides that a person who maliciously injures his neighbor in reputation or person or property should be punished. Whether John Smith, who has injured his neighbor, Richard Doe, in the matter of reputation, should be punished depends on whether his particular action comes under the general provision of the law. The argument is a deductive reasoning, which may be cast in strict syllogistic form. But when it comes to a question of the fact, the line of reasoning will be inductive, and the laws of observation together with the canons of the estimation of evidence will be employed. Every criminal trial may be represented as a syllogism in which the major premise is the law, the minor premise the alleged fact and the conclusion the verdict. The minor, however, is not established deductively but inductively.

Statistics. Let us take now, a department of inquiry which does not, strictly speaking, belong to scientific research, but is rather preliminary to it, namely, statistical investigation. Here, the process may be said to be purely inductive. *Statistics* is a methodical inquiry concerning aggregate phenomena, in history, in human society in general, in a particular state or community. The inquiry may aim at:

(1) Ascertaining aggregates of facts for purely administrative purposes; for example, the total of exports or imports, the total number of emigrants or immigrants, as reported to the government by its officials. The facts may be classified and arranged, but the purpose is always the practical one—the use to which these results are put in the government offices.

(2) Ascertaining facts and comparing them for the purpose of determining the average. The purpose is here the elimination of "chance" and the reduction of the complete facts to certain formulas which express the percentage, or degree of probability. Statistics of this kind, establishing the death rate per thousand, are furnished the insurance companies, and form the basis for determining the amount of their premiums or assessments. When the facts observed are sufficiently numerous, there will be very little variation from year to year. Setting aside the possibility of a widespread calamity, the chances of which are not reckoned, the death rate per thousand this year will be the same as it was last year.

(3) Ascertaining, comparing, classifying and tabulating facts for the purpose of ascertaining the effect of a given cause or the cause of a given effect. For example, the facts of commerce, the imports and exports, the amount of home trade, the amount of money deposited in savings banks, etc., are enumerated, classified, and tabulated, and then compared with other periods, for the purpose of showing how protective tariff affects the prosperity of a country.

Properly speaking, logic is concerned only with the third of these processes. With regard to the other two, it is sufficient to remark that the purpose of the statistician will determine his method. Facts, as facts, mean very little. To convey information, they must be interpreted, or at least arranged so as to facilitate interpretation. The statistician may not indeed misrepresent the facts, but he may, by arranging them and tabulating them in a special way, give them a meaning quite in accordance with his own convictions or prejudices. To "lie like a statistician" has become a current phrase. A ludicrous example of a misleading general summary of facts is contained in the sentence "All the horse thieves in Stockville, Texas, are Democrats," whereas, in point of fact, so are all the other inhabitants of the town. With regard to the third kind of statistical inquiry, all the rules of observation laid down in Chapter XVII should be applied. For this kind of investigation is merely observation of aggregate phenomena. Enumerations should be accurate; they should be complete whenever it is possible, and when an average is taken it should be taken from fairly normal, representative types in normal conditions; no material circumstance should be neglected, and interfering causes should be eliminated whenever it is possible to do so. The task of tabulating, diagramming, and representing the results graphically by curves, squares, or colored maps—this belongs to the technique of statistical work, and logic has merely to suggest that the laws of Division and Classification be observed.

Calculation of Probability. Closely related to statistical work is the *Calculation of Probability.* The probability of an event occurring in the future is based either on (a) the terms of the problem, when the latter is purely artificial, or (b) the frequency of occurrence in the past.

(a) If a box contains an equal number of blue and red balls, the chances that a blindfolded person will draw a blue or a red ball are equal. If the blue balls are twice as numerous as the red, the chances in favor of the blue are two to one. Since a die has six sides, the probability that any one, say four, will be uppermost is one-sixth; but the probability that when two dice are thrown two fours will be uppermost is one thirty-sixth.

(b) If we know nothing about the conditions which determine the occurrence of an event, we may calculate the probability of its recurrence from the frequency of its occurrence in the past. The formula for the probability of recurrence is $n+1$ against $n+2$. If an event has occurred four times the probability that it will occur again is as five is to six, or five-sixths.

ESTIMATION OF EVIDENCE. The *Estimation of Evidence,* as the phrase is commonly understood, includes (a) the appreciation of human testimony and (b) the valuation of circumstantial evidence.

(a) *Appreciation of human testimony.* The reliability of a witness depends on two qualities, (1) knowledge, that is, sufficient opportunity for observing the fact testified to, and (2) veracity, that is, a willingness to tell the truth. The first condition is determined both by the qualities, mental and physical, of the witness and by the circumstances. A blind man, an insane person, a person who was absent when the event occurred, are not direct witnesses to the event. A blind person is, of course, competent to testify to what he heard. The second condition, veracity, is in all cases to be presumed. A person is supposed to be willing to tell the truth if he has no strong motive for telling an untruth. In the absence of such a motive, his willingness to tell the truth is taken for granted. This does not mean that when a motive, such as hatred, jealousy, or the desire of gain, is proved to be present, the testimony is necessarily invalidated; because, if the witness is proved to be a man of integrity and strength of character, it is presumed that he disregards the unworthy motive and tells the truth. When several witnesses testify, their testimony may be either *contradictory*, *concurrent*, or *accumulated*. If there is a contradiction, the circumstances and the characters of the witnesses will determine whose testimony outweighs that of the others. When testimony is accumulated, that is to say, when several witnesses agree in testifying the same fact, the evidence in favor of the fact is stronger than if there were only one witness. Concurrent testimony has a peculiar strength: it exists when several witnesses, testifying independently, agree as to the fact, and their agreement cannot be explained by the supposition that they are in collusion or conspiracy; when, indeed, the circumstances are such that the concurrence cannot be accounted for except on the supposition that the fact really occurred.

(b) *Circumstantial Evidence* undertakes to prove the principal event, or fact, from subordinate facts incidental to the principal fact yet indicative of it. It amounts to a concurrence of circumstances, and gives certainty only when there is absolutely no other way of accounting for the circumstances except by the admission of the alleged principal fact. The valuation of circumstantial evidence includes the formation of a hypothesis and the testing of that hypothesis by the concurrent testimony of the circumstances. In a murder trial, for example, when there are no direct witnesses to the deed, the prosecution brings forward the hypothesis that the circumstances, such as the location and appearance of the corpse, the finding of blood-stained garments in the apartments of the accused, etc., cannot be accounted for except on the supposition that the accused is guilty. The defence, on the contrary, if it cannot deny the truth of the circumstances, strives to explain them on some other hypothesis.

History. In *History* the ascertainment of facts is governed by the same principles as the estimation of evidence in general and the valuation of circumstantial evidence. Contemporary witnesses come first in order of importance, next come quasi-contemporary witnesses, remote witnesses and tradition. The discovery of the laws of historical development, that is to say, the inferences drawn from the ascertained facts, is usually a matter of Induction. The method most commonly applied is the Method of Concomitant Variations (see Chapter XVIII).

Analogy. In *Analogy* we have to deal with a kind of inference which, while akin to induction, has a character of its own by which it is differentiated both from induction and from deduction. It is defined as "Inference based on similarity." If *A* resembles *B* in the possession of attributes *a, b, c, d,* and it is discovered that *A* possesses the attribute *e,* we are warranted in inferring that *B* also possesses the attribute *e.* The motto of enumerative Induction may be said to be "Many alike, therefore all alike"; that of Analogy is "Alike in many, therefore alike in all." Thus, because certain heavenly bodies resemble the earth in a great many respects, some astronomers infer that, since the earth is inhabited, those heavenly bodies are also inhabited. The force of an argument from analogy depends chiefly on (1) The number of independent resemblances between *A* and *B.* By independent is meant not connected in causation, for several resemblances traceable to one cause count for only one resemblance. (2) The lack of incompatibility between the attribute in question and the attributes, *a, b,* or *c* which *B* already possesses.

One of the most important applications of analogy is that by which we argue from natural, material, physical phenomena to the supernatural, immaterial, spiritual order. Here, the basis of the Analogy is the fact that God is the author of both orders, of the spiritual as well as the material, and the additional fact that He Himself, by using illustrations and arguments from the world of nature and of physical life, taught us to use our knowledge of Nature in the endeavor to rise "from Nature up to Nature's God."

Demonstration. In formal *Demonstration,* the process is exclusively Deductive. Demonstration literally means a "showing." It proceeds from premises which are certain, not merely probable, to conclusions which become equally certain. Direct demonstration uses positive arguments. Indirect demonstration establishes the truth of the conclusion by showing that the contradictory of the conclusion would lead to some absurdity; it is, therefore, called *reductio ad absurdum.* Direct demonstration is further divided into *a priori* and *a posteriori. A priori* demonstration proceeds from causes to effects. For example "A spherical body always casts a circular shadow: the earth is a spherical body; therefore its shadow (as seen in eclipses) is always circular." *A posteriori* demonstration proceeds from effect to cause. For instance "A body which always casts a circular shadow is spherical: the earth always casts a circular shadow; therefore, the earth is spherical."

Because the Deductive process is so well suited for demonstration it is used in the exposition or elucidation of general principles. It is used to show how general principles include more particular principles and how they are applied to singular, or concrete, facts.

LOGIC OF EVERYDAY LIFE. The logic of *Everyday Life* is partly inductive and partly deductive. Our knowledge of the world around us is built up from our own experience and the experience of others. Experience furnishes facts. From these we proceed inductively to frame general statements and formulate laws which express causes. Thus, we observe a number of instances in which boys and girls who have been carefully taught the principles and practices of their religion fall away from the observance of the laws of God and of the Church through the influence of evil companions. From these facts we infer that "Evil communications corrupt good manners (morals)." Or we may start with the general principle that "Evil communications corrupt good manners," which we receive on the authority of parents or teachers, and argue to a particular application of that principle. In the former case the process is Inductive. In the latter, it is Deductive. Or, to use another example, every boy learns from his observation of the habits of birds that "All Baltimore orioles build hanging nests." The process by which he arrives at that general conclusion from his own experience is, although he does not know it, a process of Induction. If, now, a companion tells him that in such and such a tree there is a Baltimore oriole's nest, he infers from the general principle that the nest is a hanging nest. The application of a general baseball rule to a particular instance is another example of the unconscious use of deduction. When there is a dispute on the diamond, it is a question either (1) of the existence and meaning of a rule or (2) of the fact to which the rule is said to apply. When the rule and its meaning are certain and the fact is beyond question, the rule is the Major Premise, the fact is the Minor Premise, and the "decision" is the Conclusion.

CHAPTER XXI

Method

METHOD DEFINED. Aristotle says that a small error in the beginning of the journey towards truth becomes a very great error later on, and it is self-evident that a cripple on the right road will reach his destination sooner than the swiftest runner who has taken the wrong road. This figure of speech, in which truth is represented as the end of a journey, and the searcher after truth as a traveler, gave rise to the term *Method,* which is derived from the Greek words meaning "after" and "way." Etymologically, Method is the "following after" or the "way to follow after truth." It may be defined as "A system of right procedure for the attainment of truth." In this widest sense all logic belongs to Method. More specifically, however, logic in general has to do with the validity of the different processes, the steps towards truth, as we may call them; while Method treats of the arrangement of those processes in such a way as to ensure the attainment of truth.

SYNTHETIC AND ANALYTIC METHOD. Method is of two kinds, *Synthetic* and *Analytic.* To keep up the figure of speech with which we began, the road which we call method may be from the general to the particular or from the particular to the general. The synthetic method corresponds to the deductive process: it starts with simple principles and proceeds to complex, particular, facts. The analytic method corresponds to the inductive process; it starts with complex particular facts and proceeds to simple, that is, general principles. Synthesis is a putting together, or composition; analysis is a taknig apart, or separation. The putting together and the taking apart are, of course, to be understood of the comprehension of the terms involved. Geometry, for instance, begins with relatively simple axioms and definitions and goes on to infer the properties of right-angled, or equilateral triangles, or the characteristics of a square inscribed in a circle. The process is from what is simple in comprehension to what is complex in comprehension. In chemistry, on the other hand, we begin with a concrete substance, the comprehension of which is relatively complex, and by experimental processes arrive at conclusions about "an acid", "an alkali", "a salt", which, relatively speaking, are simple in comprehension. If we pay attention to the extension, we find that while we are "putting together" the qualities and attributes which make the comprehension more complex, we are reducing the extension, taking it apart, as it were, and making it more simple. Thus, when we pass from the consideration of triangles, squares, and circles, in general, to the consideration of this particular kind of triangle, square, or circle, we are passing from what has greater to what has less, extension. And when, on the contrary, we pass from the study of this particular kind of substance to the consideration of "an acid", "a salt", etc., we are passing from what has less to what has greater, extension. Hence, it is said that Synthesis in comprehension is Analysis in extension and Analysis in comprehension is Synthesis in extension. If we place the general above the particular, thus

General

Particular

the upward is the analytic, inductive, process, an analysis in comprehension, but a synthesis in extension, while the downward is the synthetic, deductive, process, a synthesis in comprehension but an analysis in extension.

Synthetic and Analytic Sciences. As was pointed out in Chapter XX, there can be no hard and fast line drawn between inductive and deductive sciences. Still, there are, as we saw in that chapter, some sciences which are predominantly deductive and others in which induction predominates. In the same way, we may consider that in some sciences, in geometry, ethics, and metaphysics, for example, the method used is almost entirely deductive, while in other sciences, such as chemistry, biology, philology the method used is almost exclusively inductive.

Exposition and Discovery. Sometimes, these two kinds of method are described as the *Method of Exposition* and the *Method of Discovery*. Not that the synthetic, or deductive, method has no application to discovery. It has, as the dependence of many discoveries on the use of mathematical reasoning shows. But, it is true that discovery generally begins by the analytic or inductive process; and exposition or the synthetic method comes in only at the point where truth already ascertained is to be applied to particular uses, or it is desired to give a comprehensive, unitary view of the whole subject. Let us suppose that one of our Indian missionaries sets to work to study the language of a tribe among whom he is to preach the Gospel. There are no grammars, no dictionaries, no written works in that dialect. He must proceed, laboriously, to put together the facts of his own observation, he must note how the words are inflected to signify number, gender, case, tense, mood, etc., how the words are combined to form sentences, how concordance of tense, mood, etc., are observed. When this detailed study is complete he is in a position to draw up a grammar of that dialect. He has discovered the rules, and his method has consisted in analysing or "separating out" the particular facts into the elements which give the general laws. If, now, he wishes to teach a fellow missionary who knows the rules of grammar in French or English, he will not need to put the learner through all the tedious process of discovering the rules, but hands them over in their universal form and teaches the learner how to apply them. This is the method of exposition, the synthetic method, by which the grammatical elements are put together or synthesised into the concrete spoken or written language. Or, apart from the task of teaching, the discoverer of the laws of this particular Indian tribe, being a student of comparative philology, may draw conclusions from the general grammatical laws which he has discovered. Here, again, the method is synthetic, or deductive.

When it is said that the missionary may teach his fellow missionary according to the deductive method, there is no intention of implying that he must do so. Because, as we shall see, there are other methods of teaching, and the first canon of method given below will often necessitate the adoption of the opposite, or inductive, method in the teaching of language. And this shows, once more, that the terms *Method of Discovery* and *Method of Exposition* or *Instruction* are not to be taken in the exclusive sense.

Synthesis has its place in discovery, and instruction often profits by the analytic method.

Before we come to the canons of logical method, we may remark here that the Deductive Method makes more use of formal definition and formal division than does the Inductive. Since the Deductive Method begins with general principles, it starts, not with percepts, but with concepts: the images with which it deals are universal. Therefore, precision and clearness and order are, in that stage of our knowledge, to be attained, not so much by the exercise of the senses as by the accurate definition and orderly division of our mental images.

Method, unlike the validity of reasoning processes, the consistency of propositions and the clearness of ideas, cannot be made a matter of rules the observance of which would infallibly ensure it. We know absolutely that if a syllogistic argument conforms to the eight rules laid down in Chapter XIII it is valid. There are, indeed, rules of Method; but they are not easily applied to individual instances. So true is this that some logicians hold method in the highest sense to be a natural gift and not a technical system. Taste and tact and an innate sense of the fitness of things are safer guides than formal rules, when it comes to a question of method, that is, to a question of setting forth one's arguments and evidence in such a way as to produce the best effect. Nevertheless, natural ability in this direction is undoubtedly aided by general considerations such as the following:

Rules of Method. *Rule I. We should have a clear conception of the end we wish to attain.* The first requisite, if we wish to attain any object in a discourse, in an essay, or in any argumentative composition or scientific treatise, is to ascertain definitely what that is, and orientate ourselves accordingly. Just as the mariner lays his course, so the writer or speaker should place before his mind the path or road he is to follow. This knowledge of the aim or end will determine the choice between inductive and deductive reasoning, and in general, will enable one to decide many of the details of arrangement, sequence of thoughts, etc.

Rule II. The starting-point should be that which is most easily understood. Here, a distinction familiar to the schoolmen finds apt application. Some things are more easily understood because of the previous training and the content of the mind of the person to whom we are trying to make them intelligible—they are more simple, not in themselves, but in relation to us. Other things are more easily understood, not in reference to our minds or the contents of our minds, but in themselves. Thus, for us, the fact that a piece of cork floats on water is a simpler truth and more easily understood than the law that a body floats on water if it displaces more than its own weight of water. Yet, in itself, the law is more simple than the complex fact. The Rule here given refers to that which is more easily understood by the person whom we wish to instruct or convince. This applies especially to the method of teaching. Although principles are in themselves simpler than facts, we should begin with facts because, for the average mind unacquainted with a principle, the best way to make the principle intelligible is to lead the learner to discover it for himself, or, starting with the facts, to follow the line of discovery. "I am convinced," writes Edmund Burke, in the *Essay on the Sublime and Beautiful,* "that the method of teaching which approaches most nearly to the method of investigation is incomparably the best, since, not content with serving up a few barren and lifeless truths, it leads to the stock on which they grow; it tends to set the learner himself in the track of invention, and to direct him into those paths in which the author has made his own discoveries." When, from the nature of the science, it is necessary to begin with principles, as in geometry, the principles should be made intelligible by reference to the concrete facts on which they are based; concepts should, indeed, be defined in general terms, but they should be referred back to the percepts from which they are derived.

Thus, *the pedagogical order is often the inverse of the logical order.* For, the truth which naturally comes first, considering the nature and previous content of the mind, is not always the truth which should come first, logically, that is, considering the abstract relation among the truths themselves.

Rule III. Due sequence and continuity should be observed. The road to truth is not always straight. It is not laid out "as the crow flies." The logical sequence and continuity of truths must be observed. But the psychological sequence and continuity should not be overlooked. The authors of the *Port-Royal Logic* write "It is beyond all question that we learn with incomparably greater facility, and retain much better what has been taught us in the true order; because the ideas which have a natural connection arrange themselves much better in our memory and suggest each other more readily."* The important thing is to determine what is the "true" order. Sometimes logic determines the question: sometimes psychology asserts its claim, and decides that considerations should be offered in the order in which they naturally suggest one another. Thus, if one were engaged in establishing the generalization that "all cruel persons are cowards," logic would demand merely that one should begin with concrete in-

**Port Royal Logic*, p. 314.

stances, then psychology would determine that the instances from the actual experience of the audience, examples of cruel persons whom the audience knows to be cowards, should come first; next, in the historical review of examples, psychology would require that instances linked together by bonds of continuity in time, place, etc., should be mentioned in sequence. The continuity mentioned in this rule has reference to the requirement that there should be no jumping indiscriminately from one consideration to another. Of course, Rhetoric also determines the order of the thoughts presented, even in an argument. The use of the climax, for instance, may necessitate a change in the arrangement of instances or examples.

Rule IV. Definition and division should be employed wherever necessary, and conducted according to the rules of logic. The subject itself should be defined, so as to distinguish between what is essential and what is accidental. The following example is given in the *Stonyhurst Logic.* "We may have observed in the newspapers that a larger number of persons lose their lives by drowning on a Sunday than on any other day. On this fact the Scotch Presbyterian makes the remark that it can only be explained by the anger of God with all who take their pleasure on His holy day; quite overlooking the circumstance that it is on Sunday that a great number of excursionists of the lower and middle classes, who are unskilled in the use of boats and rarely can swim, take their pleasure on the water."* Here the strict Sabbatarian fails to define the problem: he overlooks what is the essential element, the vastly greater number of possible victims of drowning accidents on Sunday. The division of the topic, too, should be carefully done. The various parts of the question to be discussed should be separated off, and the subdivisions carefully made.

Rule V. The same certainty cannot be attained in all departments of knowledge. St. Thomas remarked† that there are some who, having been trained in mathematical reasoning, require mathematical demonstration even when the nature of the subject does not admit of rigorous proof. Others, he says, will not receive any truth unless it has passed the test of the senses. It is a mark, he adds, of a well-trained mind to look in every science for only that degree of certitude which the nature of the science admits.‡ This applies especially to moral and social problems, in connection with which many and complex considerations must be taken into account before one can frame a proof. The mathematical sciences are free from the task of considering complex circumstances; they are

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**Stonyhurst Logic*, p. 470.
†*In II. Metaph., Lect. V.*
‡*In I. Eth., Lect. III.*

thus enabled to reduce problems to simple formulas. To require simplicity of this kind in the more complex sciences would be contrary to sound method.

Rule VI. Attention should be paid to the correct use of terms. No term should be employed unless it is understood. If there is any doubt as to the meaning of a term, a definition should be given. If there were one term and only one for each of the leading ideas in philosophy and in the other sciences, scientific terminology would be ideally perfect. Unfortunately—and this is true especially of philosophy—there is the greatest divergence among writers in their use of the most important terms. For one writer "soul" means "mind," for another it means "the sum of all our mental states," for a third it means "the principle of life." Many English writers use the term "substance" as if it meant material substance merely. Thus, it is evident that the use of a term in a certain sense, when there are various senses in which it may be used, implies the acceptance or rejection of a doctrine in philosophy. For this reason, terms should be defined, unless the meaning of them is perfectly clear. In the natural and biological sciences the conditions are much better. Each science has built up its own terminology, and, once the meaning of a technical term is made clear, all that method demands is that the term be always used in its technical meaning.

CHAPTER XXII

Appendix: Categories and Predicables

CATEGORIES. The objects of thought, which are represented in mental images and expressed by means of terms are almost infinite in number. They are grouped together in different classes more or less extended. The supreme classes, under which all of them are included, the most extensive of all the groups of objects, are called *Categories.* When we begin to compare the objects of our thoughts, the "things" about which we speak, we find that some of them are capable of subsisting by themselves, while others are of such a nature that they require something in which to inhere. "Color," for example, must be the color of something, "size" must be the size of something, "smoothness" must be the smoothness of something. When we hear of color, size, smoothness, we are naturally inclined to ask, "The color, size, etc., of what?" It is not so in the case of the apple, the wall, the table. These are substances, the former are accidents. The world of our experience is made up of both; apples have color, the wall is large, the table is smooth. The first differentiation, therefore, of the "things" represented in our mental images is into *Substances* and *Accidents*. A *Substance* is that which is capable of existing without a subject in which to inhere. An *Accident* is that which naturally requires a subject in which to inhere.

TEN CATEGORIES. Aristotle, considering and comparing the various kinds of accidental modifications of substances, found that there are nine principal kinds, Quantity, Quality, Relation, Place, Time, Action, "Passion," Posture and Habit. These, together with Substance, are the ten Aristotelian Categories.

SUBSTANCE, as defined above, is that which is capable of existing without a subject of inherence. All bodies are substances; all created spirits, such as the human soul and angels, are also substances. It is one of the most serious defects of English philosophical terminology that "Substance" is generally used as synonymous with "body" or "material substance."

QUANTITY is the extension of a substance, its length, breadth, and depth, or, as we commonly say, its size. When the quantity is continuous, it is called *Magnitude,* as the length of a line, the breadth or thickness of a board, the cubic extent of a block of marble. When it is discontinuous, it is called *Number,* as the five houses in this block, the three Wise Men, the four seasons of the year.

QUALITY is a determination of Substance characterizing, not its extent, but its nature, as a "beautiful" house, a "white" block of marble, a "wise" man, an "interesting" book. Qualities apply to spiritual as well as to material substance. We speak of a beautiful soul, a pure spirit, a provident Deity.

Relation is the order which holds between one substance and another in such a way that one implies the other. Thus, "master" and "servant," "teacher" and "pupil," stand in relation to each other. Equality, similarity, superiority, inferiority are relations.

Place answers the question Where?—As "In New York," "in the Park," "in Church," "At Home."

Time answers the question When?—As "last year," "tomorrow," "February 20, 1909."

Action is the production of some change in another or in oneself. When the change is in oneself the action is said to be Immanent, when in some other, the action is said to be Transient. Thinking, learning, are immanent actions; speaking, teaching, are transient actions.

"Passion" is here used in a very general sense, and does not mean merely violent emotion, such as anger, but any reception of change whatsoever. It is the correlative of Action "To be taught," "to be advised," "to be honored," are instances of "passion" in this sense. Whatever is acted upon, the marble, for instance, that is being made into a statue, is said to be in a state of "Passion."

Posture means the relative positions of the parts of an object. A book that lies flat on the table is in a different posture, though practically speaking, in the same place, when it is made to stand on end. Sitting, standing, lying down, are different postures.

Habit is the determination arising from the physical adjuncts which belong in a sense to the substance, although external to it. "Caesar in his armor," "Caesar with his cloak on," "Bayard in his coat of mail" are instances of substances affected or determined by "habit."

When, in describing an object of thought, we say that it is "a tree" we assign it to the Category Substance, when we say that it is "thirty feet high" we assign that phase of its being to the Category Quantity; when we say that it is "useful," we indicate its Quality; that it "grows" refers to the Category of Action; that it "is burned" refers to "Passion"; that it is in the park assigns the place; that "yesterday it was in the forest" refers to time, etc.

The Categories are sometimes called *Predicaments,* which is merely the Latin equivalent of the Greek word "Categories" used by Aristotle.

Predicables. There is another way in which we may reduce our mental images to Supreme

Classes. Instead of classifying them according to the classifications of the things which they represent, we may classify them according to the way in which one is related to the other when it is Predicate and the other is Subject of a proposition. This mode of classification gives us the *Five Predicables.*

When a Predicate is affirmed of a Subject, either

I. It represents all the essential nature of that subject, as when I affirm of this three-sided figure that it is "a triangle." The predicate, in this case, expresses all the essential nature of the subject. This mode of predication is called the *Species.*

II. It represents, not all the essential nature of the subject, but only that part of the essential nature of the subject which is common to many, as when I affirm that the object is a "figure." It is part of the essential nature of a triangle to be a figure, namely, that part which triangles have in common with circles, squares, etc. This mode of predication is called the *Genus.*

III. It represents that part of the essential nature of the subject which is peculiar to the subject and differentiates it from others of the same genus, as when I predicate of the object before me that it is "three-sided." To be three-sided is part of the essential nature of a triangle and marks it off from circles, squares, etc. This mode of predication is called the *Difference.*

IV. It represents an attribute which, while it is not part of the essential nature of the subject, follows from, or flows from, the essential nature of the subject, as when I affirm that this figure "has the sum of its angles equal to two right angles." This attribute does not belong to the essential nature of a triangle—it is not included in the definition of a triangle. Yet, it can be shown from the nature of a triangle that every triangle necessarily possesses this attribute. This mode of predication is called *Property.*

V. Finally, the predicate may represent an attribute which is neither a constituent of the essential nature nor flows from the essential nature of the subject, but merely "happens" to belong to it, as when I affirm, "this triangle is equilateral." Here, the predicate neither constitutes the essential nature nor is a consequence of the essential nature of the subject. This mode of predication is called *Accident.*

THE FIVE PREDICABLES. There are then, five Predicables, or five supreme classes to which (1) All modes of predication may be reduced (2) All our mental images may be reduced according to the relations in which they, as predicates, stand to a certain subject. The predicables are *Genus, Species, Difference, Property,* and *Accident.*

Genus is a mode of predication in which the predicate represents that part of the essential nature of the subject which is common to many similar subjects. When we say of a human being that he is an animal, we assign the Genus. In addition to this use of the term genus in comprehension there is also the more common use of the term in extension to denote the class or group of individuals to whom the generic name may be applied. In this sense the genus "animal" denotes all the individuals, men as well as brutes, designated by the name.

Species is a mode of predication in which the predicate represents all the essential nature of the subject. The predicate in this case must include in its comprehension all the attributes and qualities which should be contained in a definition of the term used as a predicate. Thus, when I affirm of a human being that he is a "man," I express in the predicate his essential nature, and if I were to define the predicate I should include in the definition all the attributes which make a human being to be a man, namely his animal nature and his rational nature. Like the genus, the species may be taken in extension. Of course, since it contains in comprehension all the essential nature of the subject, and the genus contains only part, it is clear that in extension the species is narrower than the genus. The genus is the larger class, including the species as a lower class. Thus "animal," the genus, includes "man," the species; "figure," the genus, includes "triangle," the species, etc.

Difference is a mode of predication in which the predicate represents that part of the essential nature of the subject by which, as a species, it is separated off from other species of the same genus. It includes in its comprehension all those attributes which the comprehension of the species adds to that of the genus. Thus, if "man" is defined as a "rational animal," "animal" being the genus, and "man" the species, "rational" is the Difference. We may, therefore, set down the formula *genus + difference = species, in comprehension.*

Property is a mode of predication in which the predicate represents an attribute or attributes which neither constitute the essence nor belong to the essence of the subject but nevertheless necessarily follow from the nature of the subject. The old authors on Logic gave "risibility" as a Property of man, because they maintained that, while the power of laughter does not belong to the essential nature of a human being, it follows from the rational nature of man. The power to learn an abstract science, the ability to make a free deliberate choice, the capacity to receive supernatural grace and sanctity might also be cited as Properties of man. It is evident from the nature of Property that it signifies an attribute which belongs to all the members of the species of which it is a Property. In other words, the extension of the specific Property is the same as the extension of the species.

ACCIDENT is a mode of predication in which the predicate represents an attribute or quality which is neither part of the essential nature of the subject nor follows necessarily from the nature of the subject, but nevertheless belongs to the subject. Thus "learning," "piety," and "strength of character" are perfections, and "ignorance," "lack of piety," "weakness" are defects, which do not constitute the essential nature of man, nor do they necessarily follow from his essential nature. When they are present, they are said to be Accidents. Accidents are Inseparable or Separable. An *Inseparable Accident* is one which is found in every member of the species, and a *Separable Accident* is one which is found in some members only. Thus, the whiteness of the swan was considered an Inseparable Accident until black swans were discovered in Australia. The blackness of the crow is considered to be an Inseparable Accident.

PREDICAMENTAL ACCIDENT AND PREDICABLE ACCIDENT. Attention should be paid to the twofold meaning of the term Accident in the foregoing paragraphs. In speaking of the Categories, we said that an Accident is that which is incapable of existing without a subject in which to inhere. This is usually called the *Predicamental Accident,* and is the opposite of Substance. In speaking of the Predicables, we use the word Accident to designate a certain mode of predication in which the predicate represents a quality which neither belongs to the essence nor flows from the essence of the subject. This is, for the sake of clearness, called the *Predicable Accident.* A quality may be a predicamental accident without being a predicable accident. For instance, "rationality" and "the power of laughter" are predicamental accidents, that is to say, they are not substances. Yet, they are not accidents in the predicable sense, the former being the Difference and the latter a Property in the case of man.

Comparison of Categories and Predicables. Both the Categories and the Predicables are classifications of our ideas or mental images. They may be said to be attempts to reduce the contents of our knowledge to certain supreme heads or classes. There is, however, an important and fundamental difference between the two schemes of classification. The Categories are primarily a classification of "things," and are a classification of our ideas or mental images in so far as these represent different kinds of objects or "things." The Predicables, on the contrary, are primarily a classification of modes of predication. They exhibit the different relations in which the predicate of a proposition may stand to the subject. They are a classification, therefore, of our ideas in so far as one idea is related in judgment to another.

UNIVERSALS. Both the Categories and the Predicables are *Universals;* the former, however, are *Direct,* the latter are *Reflex* Universals. A Universal, in general, is that which, being itself one, is related to many. Thus, we speak of a Universal Cause, meaning that the cause, while remaining one, is related in causation to many effects. Similarly, an image may be Universal if, while remaining one, it represents many objects. Again, a type may be universal if, while retaining its identity as a type or exemplar, it is exemplified or typified in many effects modelled after it. Finally, an idea or mental image may be universal in two ways: (1) When, like any other universal image, it represents many objects; (2) When the mind, recognizing this universality of representation, reflects that the image in question represents something which may be predicated of many. The universality of representation is direct; the universality of predication is reflex, that is, results from the exercise of the reflexive power of the mind. Since the Categories are merely a classification of ideas according to the things which they represent, the universality of the Categories is representative, or direct. The Predicables, however, are a classification of ideas according to the different ways in which they are predicated. They are, therefore, universal in the reflex sense.

Of course, the universality of the Categories is not *merely* representative. The Categories are a classification of real things, and, as such, are universal in a metaphysical sense, which it is not necessary to explain here. So far as we are concerned, so far, namely, as the Categories are a classification of ideas, they are universal in the representative, or direct, sense, because each of them, being one mental image, represents many objects. The Predicables, besides representing many objects, are by the reflexive act of the mind referred to many subjects as predicates, and are, therefore, Reflex Universals.

First and Second Intentions. This difference between the Categories and the Predicables is sometimes expressed by saying that the Categories are *First Intentions,* while the Predicables are *Second Intentions.* The word intention must here be stripped of its usual meaning of intent or purpose. The meaning is that the mind in its first consideration of mental images regards them as mere representations—these are First Intentions. In the reconsideration, or Second Intention, the mind reflects on the mental images as predicable in a certain way of certain subjects, and thus constitutes the reflex universals which are reduced to five supreme heads in the Predicables.

Controversy Concerning Universals. During the Early Middle Ages the question which commanded the greatest amount of attention in the Christian Schools of Europe was the nature and manner of existence of Universals. The solution of the question ranged between extreme Nominalism and extreme Realism. The *Nominalists* held that Universals are only names; that there are no universal ideas and that there is nothing outside the mind (nothing real) to correspond to the universal name. The *Conceptualists* admitted that the name is universal, maintained that the idea also, or Concept, is universal, but denied that there is anything outside the mind to correspond to the universal concept. The *Realists* admitted the universality both of the name and of the concept and asserted moreover that there is outside the mind something real corresponding to these. *Exaggerated Realists* held that there are real things corresponding to our universal terms and concepts. *Moderate Realists* held that all the "things" which exist are actually individual, but that in them there is a germ, so to speak, of universality, which the mind develops into full-blown universality. The universal, therefore, they said, exists fully developed in the mind alone, but fundamentally, or germinally, it exists in the world of things around us.*

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*For the history of this important controversy Cf. Turner, *History of Philosophy*, pp. 265 ff, and 352, 353.

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